NUCLEAR PHYSICS   7 classes

Further study is required in this course. Evolving field. Gets maximum Nobel Prizes.

1) Basic Nuclear Properties

Size
Constituents - their properties
Angular Momentum
Magnetic Moment
Quadrupole Moment
Parity
Binding Energy

2) Models of Nucleus

① Semi Empirical Model (Calculates Binding Energy)
Mass Formula
Mass Parabola

② Shell Model (Calculates rest of properties)

3) Nature of Nuclear Force

Characteristics of strong nuclear force
Yukawa's Meson Theory
Grand state of deuteron & Magnetic Moment

4) β decay

Parity Violation

⑤ γ-decay

Internal Conversion

Mossbauer Effect

⑥ Nuclear Fission & Fusion

Particle Physics

3 classes

① Particle Classification

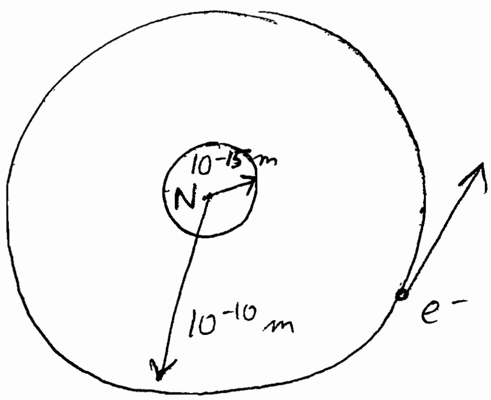
② Conservation laws ③ Quark Structure Hadrons

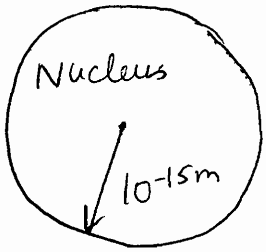
④ Basic idea about unification

Tayal / Pandey

NUCLEAR PHYSICS (1)

→ Remember α- particle experiment by bombarding them on a Gold Foil. It led to discovery of Nucleus by Rutherford. By H.U.P., we now know e^- cannot reside in nucleus. Neutron was discovered muca later, to solve **mass discrepancy**.





1 fm : 10^{-15} m

[femto meter or fermi meter]

① Constituents of Nucleus

(For proton, expected $\mu = 1\,\mu_N$ ⇒ Non uniform charge distribution)

Proton : $q = +e$

$m_p = 1836\, m_e \simeq 1.66 \times 10^{-27}$ kg

$m_0 c^2 = 938$ MeV

$I = s = \left(\frac{1}{2}\right)$ $\quad I^2 = \frac{3}{4}\hbar^2$

↑ used for nucleus ↑ used for e^-

$g \cdot \frac{e}{2m_p} \frac{1}{2}\hbar = \frac{g\mu_n}{2} = \frac{5.586}{2}\mu_n = 2.793\,\mu_n$

$(\mu_p)_z = \pm\, 2.793\,\mu_N \approx +2.8\,\mu_N$

Neutron $q = 0$

$$m_n = 1840\, m_e \simeq 1.67 \times 10^{-27}\ kg$$

$$m_0 c^2 = 939.5\ MeV \simeq 940\ MeV$$

$$(\mu_N)_Z = \mp 1.91\, \mu_N \approx -1.9\, \mu_N$$

(inspite of the fact that it is uncharged ∴ expected $\mu = 0$)

[Note that it is μ_Z that we are talking about]

$$I = \left(\frac{1}{2}\right)$$

$$I^2 = \frac{3}{4}\hbar^2$$

We represent Nucleus as (Z, A)

↑ Z: atomic number

↑ A: mass number

No. of Protons $= Z$

No. of Neutrons $= A - Z$

Special Types [redacted] of Nucleus :

[redacted] : Z same , A difference — same element

[redacted] : Z different , A same — different elements

[redacted] : [redacted] Neutrons and Protons are interchanged.

& $N - Z = 1$

$A - 2Z = 1$

$^{15}_{7}N$ unstable nitrogen

$^{15}_{8}O$ unstable Oxygen

$^{3}_{1}H$ (tritium)

$^{3}_{2}He$

Mass of Nucleus = Mass of Z protons $(Z m_p)$

+ Mass of (A − Z) neutrons $((A-Z) m_N)$

− Mass equivalent of Binding Energy $\left(\frac{E_B}{c^2}\right)$

here E_B is taken as positive value i.e. $|E_B|$

→ In any atom, more than 99% mass is contributed by nucleus.

$$\Rightarrow \frac{E_B}{c^2} = [Z M_p + (A-Z) M_N] - M(Z,A)$$

[representation of mass of nucleus]

$$E_B = \left(-M(Z,A) + [Z M_p + (A-Z) M_N]\right) c^2$$

difference in a.m.u & m_p/m_n due to B.E

due to "Mass defect", we have **1 a.m.u. = 931 MeV** < m_p or m_n

Mass of Constituents − Mass of Nucleus : Mass defect

Binding Energy per nucleon = Average Binding Energy

Nuclei is written in 4 ways:

Pairing [N − Z]	Mass No. A	No. of stable nuclei observed
Even − Even	A even	165
Odd − Odd		4
Even − Odd	A odd	55
Odd − Even		50
N Z		

→ $^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}B$, $^{14}_{7}N$ (deuterium) $N = Z$

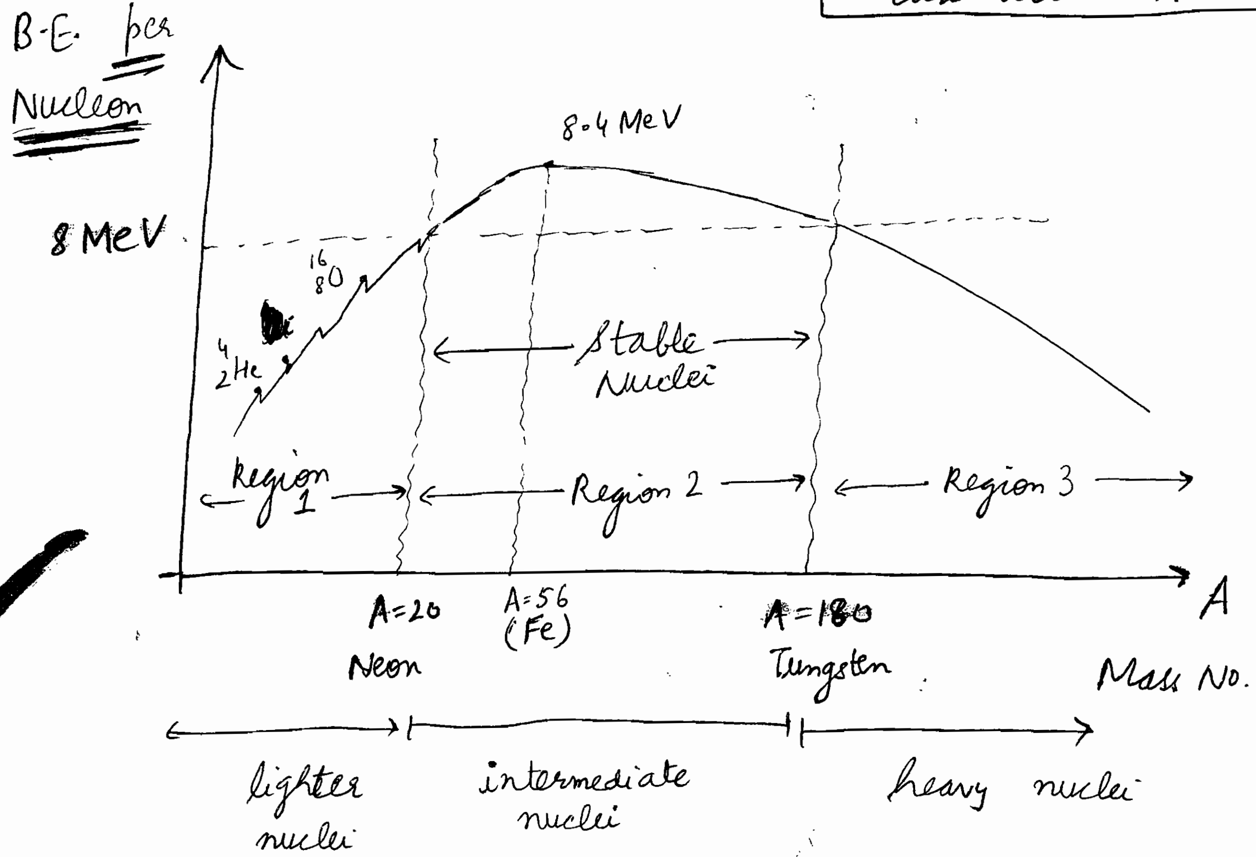
≃ 117 elements ≃ 270 nucleus (stable)

[Many stable isotopes]

- Even - Even are most abundand ⇒ most stable
- Odd - Odd are least abundant ⇒ least stable
- Even-Odd / Odd -Even are normal.

② 'Binding Energy per Nucleon' Curve

> Note that in nuclear physics, we are least interested in Z. We care about A.



It is an empirical (experimental) curve.

→ More the binding energy, more the energy needs to be supplied to break it ⇒ more the stability.

→ Nuclei of [redacted] group1 fuse together to gain higher binding energy to achieve stability: Nuclear Fusion

→ Nuclei of group 2 break to come to region 2, to increase Binding energy in order to achieve stability.

Nuclear Fission & Spallation

(*) More stable the nuclei, more Binding Energy will be released ⇒ reason for energy in fusion & fission

Higher the mass, higher the volume

$\Rightarrow$ High A [redacted] nucleus will have more Volume

$\Rightarrow$ more radius (if assumed spherical)

$$\left(\frac{4\pi}{3} r^3\right) \propto A$$

$$\Rightarrow \boxed{r \propto A^{\frac{1}{3}}}$$

$$\boxed{R = R_0 A^{\frac{1}{3}}}$$

$\uparrow$

Radius Const. 1.2 - 1.3 fm

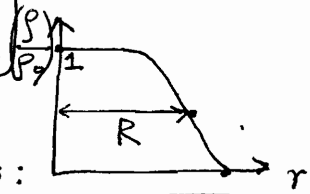
Take $r_0 = 1.3$ fm

⊙ The size and shape of neuclei are studied by scattering experiments using high speed electrons and neutrons as Bombarding Particles.

Electrons interact only with protons and neutrons interact only with special nuclear forces.

The former provides information on distribution of electric charge and the latter on the distribution of nuclear matter in the nucleus.

(*) ρ is distribution of charge in the nucleus:



$$\text{Density } \rho = \frac{A \times 1.66 \times 10^{-27} \text{ kg}}{\frac{4\pi}{3} R_0^3 . A} \simeq \frac{10^{-27}}{10^{-45}}$$

$$\simeq 10^{17} \text{ kg/m}^3$$

Now we see density is independent of size or shape as in real world. Similarly is the case with nucleus. Density of Nucleus is independent of mass.

Hence, Nucleus is compared to a liquid drop

eg. (1) o o $\rightarrow$ O Fusion (Coalescence)

(2) O $\rightarrow$ o o Fission (breaking down)

(3) ρ independent of shape & size

(4) Both are spherical in shape.

Both neutrons and protons have spin $= \left(\frac{1}{2}\right)$ and possess

(3) angular momentum.

Hence L-S coupling occurs within nucleus.

$$\vec{I} = \vec{L} + \vec{S} \qquad \text{(L-S coupling)}$$

$$\vec{L} = \sum (\vec{l_p} + \vec{l_n})$$

$$\vec{S} = \sum (\vec{S_p} + \vec{S_n})$$

Nucleon: Neutrons & Protons are together called Nucleons

$$I = I_p + I_s \qquad \text{(I-I coupling)}$$

$$|I| = \sqrt{I(I+1)}\ \hbar \qquad \text{Magnitude Quantization}$$

I: Nuclear Spin Quantum Number

[Total ang. momentum of Nucleus it may be due to orbital or spin or both. Note that I am not interested in individual momentums I am interested only in total I]

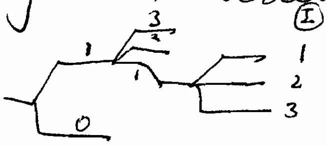
$$I_Z = M_I \hbar \qquad \text{space Quantization}$$

Here space Quantization will lead to division of energy levels. Order of $\Delta E \simeq 10^{-8}$ eV (NMR)

(*) {Here "inverted coupling" occurs b'coz now μ is positive quantity. ⇒ higher I, lower energy}

Nuclear LS coupling is inverted LS coupling i.e. [Higher I, lower energy] ✓ (★)

eg.



(4) Magnetic Moment

$$\vec{\mu} \propto \vec{I}$$

$$\vec{\mu} = \underbrace{\frac{e}{2m_p}}_{\text{Gyromagnetic Ratio}} g \vec{I}$$

g : Land 'g' splitting factor

$$I = L + S$$

$$g = 1 + \frac{I(I+1) + S(S+1) - L(L+1)}{2I(I+1)}$$

[As always l*** को negative कर दो !!]

in L-S coupled state only....

→ 'g' can be obtained from NMR.

$$|\vec{\mu_z}| = \frac{e}{2m_p} g |\vec{I_z}| = \mu_N g M_I$$

$$\boxed{\mu_z = \mu_N g M_I}$$

→ Raabi's method to measure μ of neutron

(★) To measure μ of proton, we can use simple NMR of ~~[scribbled out]~~ water.

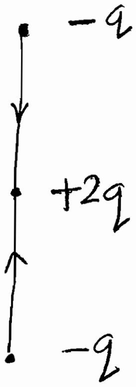
Q Why neutron has magnetic moment ??

It is not explained classically.
It is due to internal structure of neutron. Overall it is electrically neutral; it is surrounded by cloud of π^- mesons.

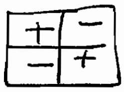
(5) Quadrupole Moment

[nuclei do not have dipole moment but have a quadrupole moment]

Quadrupole : arrangement of 4 particles



Or



In EM, $q l^2 (3\cos^2\theta - 1) \ldots$

$\rho(r)$: charge per unit volume at distance r

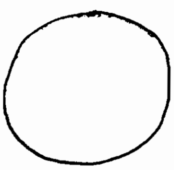
(*) For quadrupole moment symmetric about z axis

$$Q = \frac{1}{e}\int (3z^2 - r^2)\, \rho(r)\, d\tau$$

It has dimension of Area

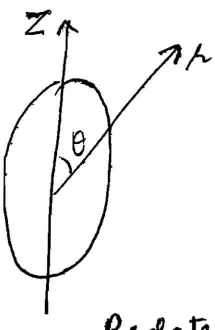
Measure in $10^{-28}\ m^2$

Area of $10^{-28}\ m^2$: 1 Barn



$Q = 0$ if charge distⁿ is spherical

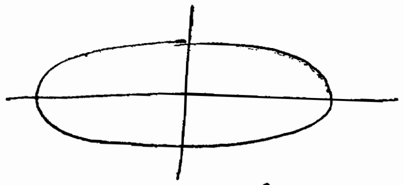
otherwise $Q \neq 0$



$Q > 0$

$$Q = \frac{1}{e}\int (3\cos^2\theta - 1)\, r^2 \rho(r)\, d\tau$$

Prolate spheroid [Rugby Ball]



$Q < 0$

(*) इसका Q negative है.

Oblate spheroid [like Earth]

[redacted]

We can assume, Nucleus fluid to be in shape of Prolate Ellipsoid or Oblate Ellipsoid.

Quadrupole Moment is the departure from spherical shape of nucleus, i.e. spherical charge distribution

It is measured in area in units of Barns.

(6) Parity:

Behaviour of particle or system or phenomenon under mirror symmetry. (inversion of coordinates)

If same behaviour ⇒ ~~Symmetric System~~ Even Parity

If opposite behaviour ⇒ [redacted] Odd Parity

If $\psi(x,y,z) = \psi(-x,-y,-z) \Rightarrow$ Even Parity

If $\psi(x,y,z) = -\psi(-x,-y,-z) \Rightarrow$ Odd Parity

$P[\psi(x,y,z)] = +1\ \psi(x,y,z)$ Even Parity

where P opposes the coordinates

if $P[\psi(x,y,z)] = -1\ \psi(x,y,z)$ Odd Parity

±1 are eigen value of 'P' operator

Note that $P^2[\psi(x,y,z)] = +1\ [\psi(x,y,z)]$

P^2 has only 1 eigen value i.e. +1

उल्टे का उल्टा : सीधा

for further reference, refer to किताब

If even parity ⇒ no change in behaviour in mirror image.

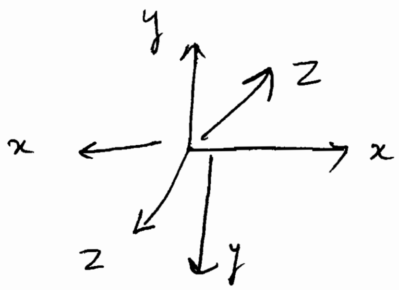
Typical example of even parity wave function : $\cos x$

odd parity wave function : $\sin x$

- Parity is a multiplicative quantity.

<u>Space Parity</u> : $+x, +y, +z \longrightarrow -x, -y, -z$

'Right-Handed System' to 'Left Handed System'



$(x, y) \rightarrow (-x, -y)$: <u>Translational Symmetry</u>

if also $(z) \rightarrow (-z)$: <u>Rotational Symmetry</u>

Every system has 2 parities : Intrinsic Parity
Space Parity

For any particle

$$\boxed{P_{Total} = P_{space} * P_{Intrinsic}}$$

For any reaction, $A + B \rightarrow C + D$

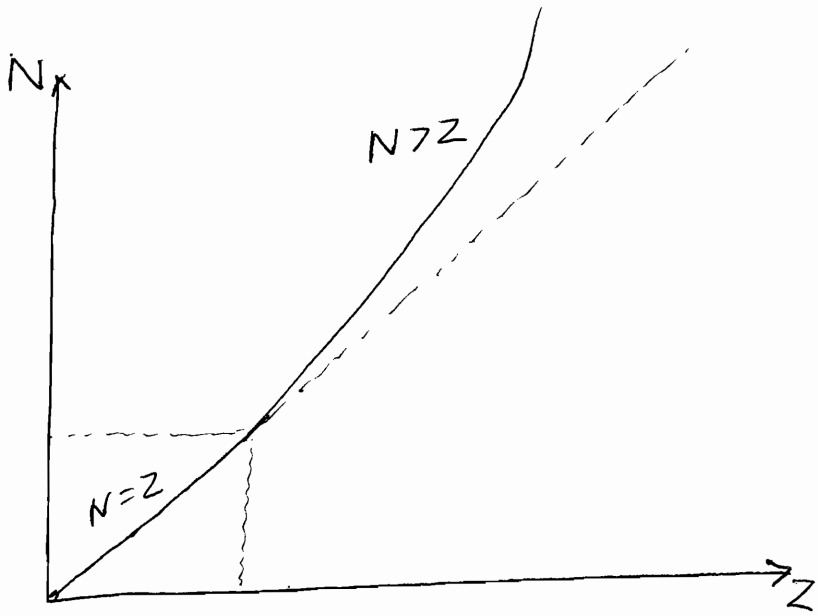
We have to see what type of phenomenon is there.
Depending on type of phenomenon, Parity will either be conserved or reversed.

$$P_{(A)} \; P_{(B)} = P_{(C)} \; P_{(D)}$$ **Parity** Conservation

If Parity is [redacted] conserved ⇒ reaction recording is independent of coordinate system whether left handed or right handed.

Stability Curve

Most stable nuclei for lighter nuclei :- No. of protons = No. of Neutrons



All nuclei with $Z > 83$ and $A > 209$ are unstable

For heavier nuclei :- No. of Neutrons > No. of Protons to achieve stabily ∵ Coulumb force of repulsion increase due to more Z. Shielding of n is required to achieve stability

~~Magnitude wise, larger [redacted] the packing, more stable is the nucleus~~

Packing Fraction

$$\text{Packing Fraction} = \frac{M - A}{A} = \left(\frac{M}{A}\right) - 1$$

→ ~~Negative quantity. Used to measure Binding Energy as a fraction of Total Energy contained in nucleus ($= m.c^2$)~~

M: mass of nucleus
A: mass no. of nucleus

here A is actually [mass number * 1 a.m.u]

Interactions

① [redacted]

due to mass

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

No use in Nuclear Physics.

Its acting on all particles, but magnitude is less.

② [redacted]

due to charge

Electrostatic Force: $F = -\frac{kq_1q_2}{r^2}\hat{r}$

Again it acts on all particles that are charged.

Rest 2 interactions are there in Nuclear Physics only:

They are dependend neither on charge nor on mass.

③ [redacted]

These are interaction b/w point particles that do not occupy any space. These particles are called LEPTONS. Range of interaction $\approx 10^{-17}$ m

} Neither dependent on charge nor on mass.

[redacted]

Leptons: $\begin{bmatrix} e^-, & \nu_e, \\ \mu^-, & \nu_\mu, \\ \tau^-, & \nu_\tau \end{bmatrix}$

They are responsible for decay reactions.

eg. β decay, muonic decay

~~Strong Interaction~~ : eg. Force that keeps nucleons together in nucleus.

It is also not due to mass or charge or decay.

In production of certain heavier particles called strange particles, strong interactions play a role.

It is the strongest known force in nature.

Symmetry Statistics of Nucleus

The quantum mechanical description of a system with a no. of particles like the nucleus is given by either Bose-Einstein statistics or Fermi-Dirac Statistics.

Bose Einstein All particles with integral spin or zero spin obey Bose Einstein (B.E.) statistics and are called Bosons eg proton, deuteron. All nuclei with even A obey B.E. statistics. Wave Function of a system obeying BE statistics is symmetric. This means that the wave function for such a system remains unaltered by interchange of the coordinate for any pair of identical particles.

$$\psi(x_1, x_2, \ldots x_i \ldots x_j \ldots x_n) = \psi(x_1, x_2, \ldots x_j \ldots x_i \ldots x_n)$$

Fermi Dirac All particles with half integral spin obey Fermi Dirac (F.D.) statistics and are called Fermions eg. e^-, proton, neutron. All nuclei with odd A obey Fermi Dirac statistics. All fermions obey Pauli's exclusion Principle. Wave Function of a system of particles obeying F.D. statistics is anti symmetric. This means that if the coordinates of any pair ~~of~~ identical particles are interchanged in the wave function, the new system will be identical with the original except for a change of sign in the wave function.

$$\psi(x_1, x_2, \ldots x_i, \ldots x_j \ldots x_n) = -\psi(x_1, x_2, \ldots x_j, \ldots x_i \ldots x_n)$$

~~Nuclear Physics (2)~~ 1/03/2012

THE TABLE : Particle Physics (1)

Type of Interaction	Particles Involved	Range	Lifetime of Interaction	(Particle Exchanged	Non-dimensional Coupling Const.	Relative Strength	Characteristics
Strong	HADRONS (Quarks) Colour Field	10^{-15} m	10^{-23} s	Gluons or Pi Mesons (in case of Nuclear Forces)	$\left(\frac{g_f^2}{4\pi\epsilon_0\hbar c}\right) \backsimeq \left(\frac{14}{[illegible]}\right)$	$\frac{14}{14} = 1$ This is reference	→
EM interaction	ALL CHARGED PARTICLES	∞	$\leq 10^{-14}$ s	Photons	$\frac{e^2}{4\pi\epsilon_0 \hbar c} = \left(\frac{1}{137}\right)$	$\frac{\left(\frac{1}{137}\right)}{14} \simeq 10^{-4}$ i.e. 10000 times weaker than Strong Force	we know
Weak	LEPTONS eg. $[e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau]$	10^{-17} m	$\geq 10^{-8}$ s	Intermediate Vector BOSONS $W^\pm, Z^0$	$\left(\frac{m\pi c}{\hbar}\right)^4 \left(\frac{g_f}{\hbar c}\right)^2$ Its called COMPTON WAVELENGTH	10^{-14}	→
Gravitational	All Particles	∞	—	GRAVITON	—	10^{-40}	we know

(*) Ironically, weak force is not the weakest force, but rather gravitational force is the weakest force.

Characteristics / Salient features of strong nuclear force:
(covered in nuclear)

(1) Interaction is charge independent
⇒ same b/w n-n, n-p etc.

(2) Attractive in nature

(3) Spin dependent

(4) Short Range. Within nuclear dimension

(5) Satisfies saturation property.

(6) Non-Central in nature i.e. Tensor force

Field of Strong Forces is called "Colour Field".
What "Charge" play role in EM field, "Colour" plays same role in Strong Field.

eg. Strong Nuclear Force
Production of strange Particles

Weak Force

eg.
β-decay
Decay of Strange Particle

⊙ Strong Force is same in $n-n$
$n-p$
$p-p$

It is explained in terms of exchange theory.

→ Field is a force per unit quanta, this we know. It is the sphere of influence of that force.

How the force field is created ??

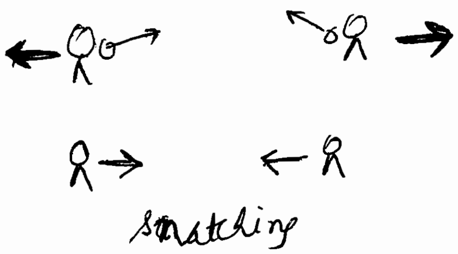
Yukawa's Exchange Theory

Strong Interaction is a broader term whose subset is Nuclear Force. For Nucleus Force, Yukawa gave Exchange Theory or Meson Theory.

It explains nature of nuclear force between $n-n$, $n-p$ and $p-p$.

According to the theory, interaction is arising due to exchange of particles [redacted]

2 players playing some game of exchanging balls.



⇒ repulsive

⇒ attractive

✪ If exchange particle of high mass ⇒ short range of interaction

$$m \propto \frac{1}{R}$$

π^0 ← exchange particle

$n \; \overset{\pi^0}{—} \; n$ ← interacting particles

$n \; \overset{\pi^\pm}{—} \; p$

$p \; \overset{\pi^0}{—} \; p$

Range of strong Force = $\underline{1.7\ F \text{ to } 2.2\ F}$

(Short Range)

I can fix up mass of exchange particle via H.U.P.

$\Delta E \, \Delta t \geq \hbar$

use $\boxed{\Delta E \cdot \Delta t \approx \hbar}$

$m_\pi c^2 \frac{R}{c} \geq (\hbar)$

$$\boxed{m_\pi \geq \frac{\hbar}{Rc}}$$

1.4 fm

Range of m_π : 235 m_e to 270 m_e

if we take Range of R : 1.7 F to 2.2 F

→ Mesons are 0 spin particles, without charge if π^0, & charge $\pm q$ if $\pi^\pm$ depending upon equation.

$n \rightarrow n$ Charge $\pi^0 = 0$

$n \rightleftharpoons p$ Charge $\pi^\pm \neq 0$

This force satisfies saturation property. One particles interacts only with its immediate neighbour. After interacting with its neighbour, it gets saturated. No further interaction.

i.e. snatching of balls from immediate neighbours only.
(जितने मरजी बातूनी हो, कितनों से बात कर लोगे तुम !!)

Specific Case

In nuclear force, nucleons interact via π mesons.

General Case

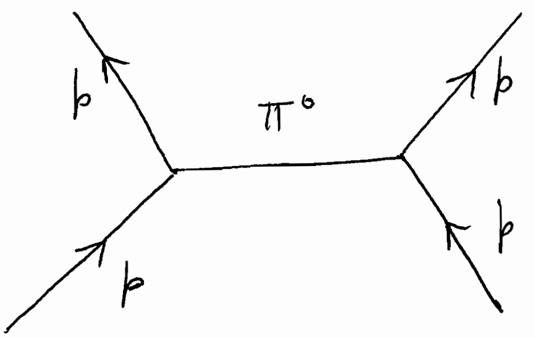
✓ In strong force, quarks interacts via gluons.

✓ In weak field, leptons interact via intermediate vector bosons.

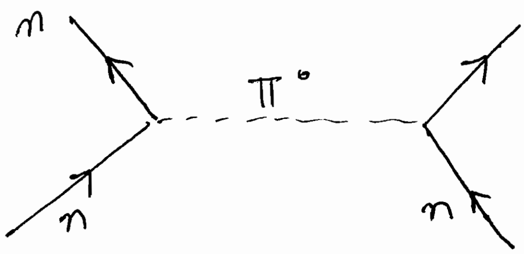
✓ In gravitational field, particles interact via gravitons

✓ In EM field, charge interact via photons.

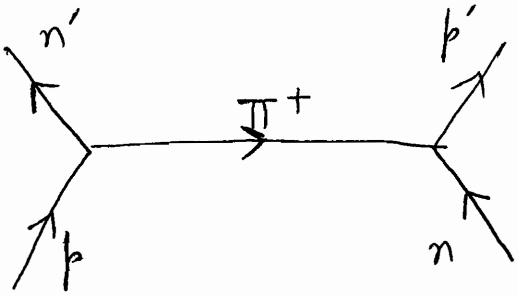
[redacted]



$p \xrightarrow{\pi^0} p$

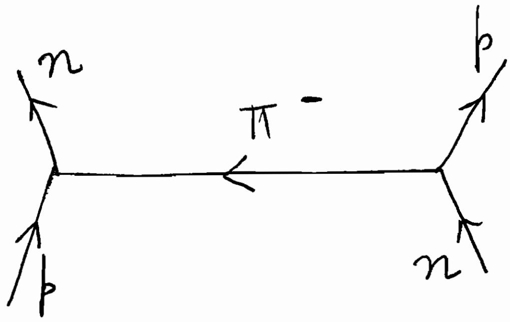


$n \xrightarrow{\pi^0} n$

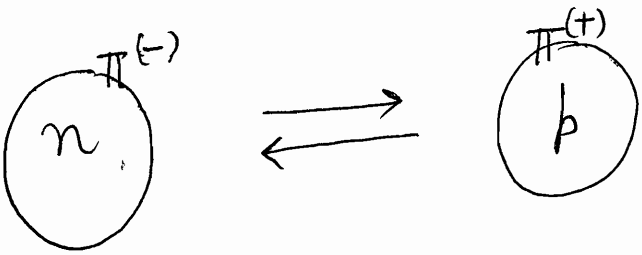


Proton आया, π(+) दिया, छूटे n' बन गया!!
Neutron आया, π+ accept किया, p' बन गया!!

- Flow of π^+ is equivalent to
- Flow of π^- in opposite direction



From the uncertainty principle in the form
$$\Delta E \Delta t \geq (\hbar/2)$$
an event in which an amount of energy ΔE is not conserved is not prohibited so long as the duration of the event does not exceed the Δt. This condition lets us estimate mass of pion



Neutrons are surrounded by $\pi^{(-)}$ mesons

Protons are surrounded by $\pi^{(+)}$ mesons

$$n + \pi^{(+)} \longrightarrow p$$

$$p + \pi^{(-)} \longrightarrow n$$

Note that Mesons have 0 spin. (according to Conservation law)

Now since neutrons are totally neutral but due to being surround by negative $\pi^{(-)}$ mesons $\Rightarrow$ they have negative magnetic moment.

Particles & Anti Particles

✓ **Elementary Particles** or Fundamental Particle: Particles whose internal structure cannot be written in terms of other particles are fundamental particles.

✓ Every particle has an antiparticle.

Particle & Anti Particle have same mass, spin and lifetime. (Note that lifetime is defined for unstable particles)
But they have opposite charge & opposite orientation of spin wrt. magnetic moment.

✓ Whenever particle and antiparticle meet, they annihilate and energy equal to $2mc^2$ is released.

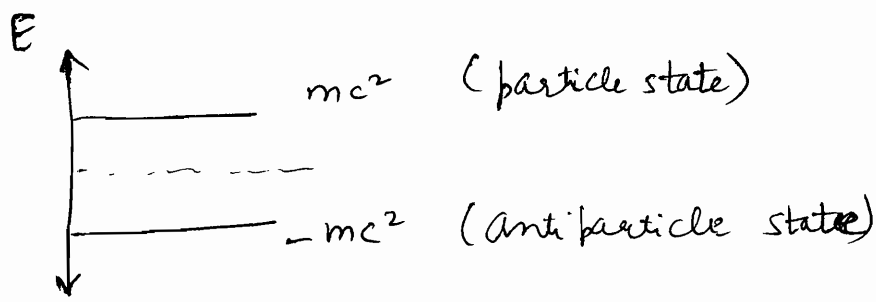
$$e^- + e^+ \rightarrow \text{Energy released}$$

electron positron

Concept of Anti Particle is a consequence of Dirac's Quantum Mechanic theory (Dirac's equation is a synthesis of Quantum theory & relativity)

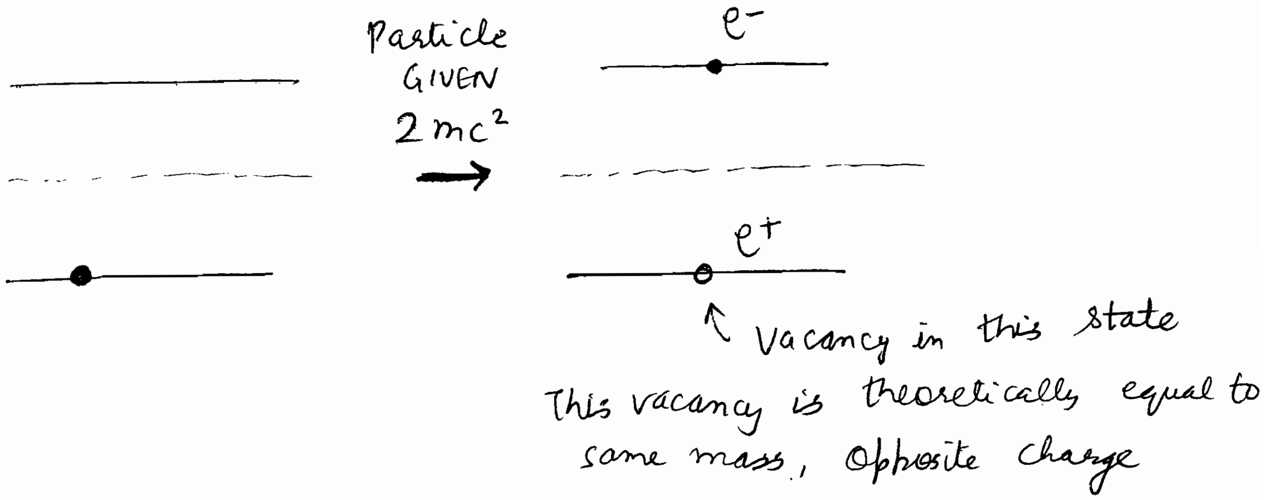
$$E^2 = (pc)^2 + (m_0c^2)^2$$

$$E = \pm\sqrt{(pc)^2 + (m_0c^2)^2} = \pm mc^2$$



[Called dirac sea, Containing negative energy state particles. But all negative energy states are fully occupied. Note that नीचे ये electrons होते हैं!!]

→ If $(-mc^2)$ state is given $(2mc^2)$ energy, it will reach $(+mc^2)$ state. Hence, anti particle becomes particle.



All particles have antiparticles except π^0, η^0 mesons.

π^0 and η^0 have π^0 and η^0 only as anti particles.

Particle Classification

4 criteria of classification :

(1) <u>On basis of spin</u> : Only 2 types of particles viz.

(i) Fermions — Half Integral Spin

(ii) Bosons — Full Integral Spin

(2) <u>On basis of interaction</u> :

(i) Leptons : weak interaction

(ii) Hadrons : strong interaction

(3) On basis of mass ↓

(4) Gellmann Nishijima Classification

(i) Strange
- ✓ Produced by strong force, ~~weak~~ decay by weak interactions
- ✓ have strangeness quantum numbers.

(ii) Non Strange
- No strangeness quantum number.

- In the standard model, there are 6+6 =12 types of elementary fermions : 6 Quarks and 6 leptons.
- Elementary particles are classified according to spin as fermions ($\frac{1}{2}$ integral) and Bosons (integral)

Classification on basis of mass

CLASS	Particles	A.P.	Spin	Lepton Quantum Number (S)	I_3
Lepton	Electron Neutrino ν_e	$\bar{\nu}_e$	(1/2)	X	X
	Muon Neutrino ν_μ	$\bar{\nu}_\mu$	(1/2)		
	Tau Neutrino ν_τ	$\bar{\nu}_\tau$	(1/2)		
	electron e^-	e^+	$(\frac{1}{2})$		
	muon μ^-	μ^+	$(\frac{1}{2})$		
	tau τ^-	τ^+	$(\frac{1}{2})$		
HADRONS (made of Quarks): [redacted]	π: π^+, π^0, π^-	π^-, π^0, π^+	0	0	1, 0, −1
	η: η^0	η^0	0	0	0
	K: K^+, K^0	K^-, $\overline{K^0}$	0	+1	$\frac{1}{2}$, $-\frac{1}{2}$
[redacted]	Nucleons P, N	$\bar{P}$, $\bar{N}$	$(\frac{1}{2})$	0	$\frac{1}{2}$, $-\frac{1}{2}$
	Hyperons λ: λ^0	$\bar{\lambda}_0$	(1/2)	−1	0
	Σ: Σ^+, Σ^0, Σ^-	$\bar{\Sigma}^-$, $\bar{\Sigma}^0$, $\bar{\Sigma}^+$	(1/2)	−1	1, 0, −1
	Ξ: Ξ^0, Ξ^-	$\bar{\Xi}^0$, $\bar{\Xi}^+$	(1/2)	−2	$\frac{1}{2}$, $-\frac{1}{2}$
	Ω^-	Ω^+	(3/2)	−3	0

↓ increasing mass

Mass के हिसाब से misfit → τ^-, τ^+

Stable Particles

(★) Only π and η mesons are exceptions that their antiparticles are among themselves. π^0, η^0 : self antiparticle

(★) Note that for Ω^-, spin $=(\frac{3}{2})$

In addition to these, Hadrons [redacted] also include resonance particles whose lifetime < $10^{-2[illegible]}$ sec electronic neutrino $\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}$ electron : +1 Leptons are $(\frac{1}{2})$ spin particles : fermions

electronic lepton quantum number $L_e = 1$

$\begin{bmatrix} \bar{\nu}_e \\ e^+ \end{bmatrix}$: −1 electronic lepton quantum number $L_e = -1$

muonic neutrino $\begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}$ muon : 1 muonic lepton quantum number $L_\mu = 1$

$\begin{bmatrix} \bar{\nu}_\mu \\ \mu^+ \end{bmatrix}$: −1 muonic lepton quantum no. $L_\mu = -1$

For rest of particles $L = 0$

tau neutrino, tau $\begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}$: +1 tauonic lepton quantum no. → $L_\tau = 1$

$\begin{bmatrix} \bar{\nu}_\tau \\ \tau^+ \end{bmatrix}$: −1 → $L_\tau = -1$

In all allowed nuclear reactions, lepton number have to be conserved individually i.e. L_e, L_μ, L_τ must be conserved

✓ Mesons are 0 spin particles : bosons
They do not have any quantum numbers.

✓ All Baryons are assigned quantum no. +1 $B = 1$
Anti Baryons assigned : −1 $B = -1$
All other: Baryon Quantum No. = 0 $B = 0$
Baryon No. should be conserved for all Nuclear Reactions.

Conservation Laws

Every allowed nuclear reaction has to satisfy following 8 conservation laws:

(1) L_e
(2) L_μ
(3) L_τ
(4) Baryon Quantum Number 'B'
(5) Conservation of charge
(6) Conservation of Mass-Energy
i.e.
Conservation of relativistic energy

[redacted] : Amount of energy released or given to a reaction so that it proceeds.

Q value of any reaction is consequence of Conservation of relativistic energy.

$$X + x \longrightarrow Y + y + Q$$

$$X(x, y)Y$$

$\boxed{Q \text{ value}}$ = k.E.(products) − k.E.(reactants)

= (Rest mass energy of reactants)

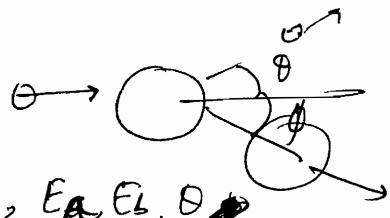
−

(Rest mass energy of products)

$k = E - m_0c^2$
↑
conserved!!

$$A + a \longrightarrow B + b$$

$$Q = E_b + E_B - E_a - E_A^{\nearrow 0}$$



replace E_B and find Q in terms of m, E_a, E_b, θ

(7) Conservation of linear Momentum

(8) Conservation of Angular Momentum

i.e.

Conservation of Spin

✓ Generally 6 are checked : linear Momentum & Mass are difficult to check

Note that they 8 aree conservation laws for weak force, strong force or EM force causing nuclear reaction.

	Strong	EM	Wea
(9) Strangeness Quantum Number "S" S is conserved in Strong & EM reactions S is not conserved in Weak field reactions.	✓	✓	✗
(10) Isotopic Spin Component "I_3" Conserved in Strong & EM.	✓	✓	✗
(11) Hyper charge $Y = S + B$ Conserved in Strong & EM.	✓	✓	✗
(12) Isotopic Spin "I" Conserved in Strong Only	✓	✗	✗
(13) C charge conjugation	✓	✓	✗
(14) P parity	✓	✓	✗
(15) T time reversal	✓	✓	✓
(16) CP	✓	✓	✗
(17) CPT	✓	✓	✓

Nuclear Physics (2)

Parity related to ⊕ve & ⊖ve Coordinates .....

Representation of Parity $\quad I = \frac{1}{2}^{(+)} \qquad I = \frac{1}{2}^{(-)}$

⊛ Intrinsic: Parity of a whole system is the product of parities of individual particles. A system having <u>even</u> no. of odd parity particles and any no. of even parity particles will have <u>even</u> parity. Similarly <u>odd</u> no. of odd parity particles ⇒ <u>Odd Parity</u>

- Photon can have any parity depending upon how created
  - Electric Interaction → ~~even~~ odd $\quad$ for $\Delta l = 1$
  - Magnetic Interaction → ~~odd~~ even

⊛ <u>Space Parity</u> = Parity of wave function that describes the coordinates of its constituent particles

- Parity of any nucleus is due to its nucleons

⊛ Parity = Intrinsic * Space $= P_\psi P_1 P_2 P_3 \ldots P_n$

- [redacted] Neutrino, Neutron, Proton : Even Parity; π Meson : Odd Parity

---

Symmetry related to interchange of particles .....

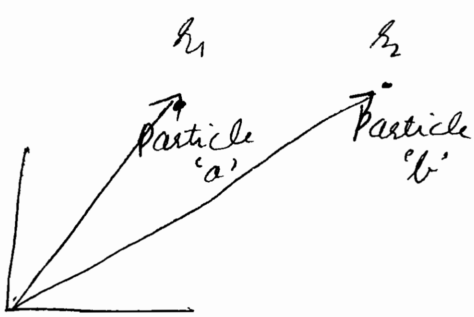
- <u>Statistics</u> : depending on
  - <u>Fermi Dirac : Antisymmetric Particle</u>
  - <u>Bose Einstein : Symmetric Particle</u>

$\psi_a(r_1)$ $\quad \psi_b(r_2)$

a at r_1 $\quad$ b at r_2

$\psi_b(r_1)$ $\quad \psi_a(r_2)$

b at r_1 $\quad$ b at r_2



Hence $\psi = \frac{1}{\sqrt{2}}\left[\psi_a(r_1)\,\psi_b(r_2) \pm \psi_b(r_1)\,\psi_a(r_2)\right]$

(+ : <u>Symmetric Wave Function</u>; − : <u>Antisymmetric Wave Funct.</u>)

$$\psi_S(r_1, r_2) = \frac{1}{\sqrt{2}}\left[\psi_a(r_1)\,\psi_b(r_2) + \psi_b(r_1)\,\psi_a(r_2)\right]$$

Bose-Einstein — Symmetric

∴ Boson are described by Symmetric Wave Functions
i.e. $\psi_a\psi_b = \psi_b\psi_a$

$$\psi_A(r_1, r_2) = \frac{1}{\sqrt{2}}\left(\psi_a(r_1)\,\psi_b(r_2) - \psi_b(r_1)\,\psi_a(r_2)\right]$$

Fermi Dirac — Antisymmetric

Antisymmetric Function gives Pauli Exclusion Principle.

∴ Fermions are described by AntiSymmetric Wave Functions
i.e. $\psi_a\psi_b = -\psi_b\psi_a$

$$\psi_A(r_1, r_1) = \frac{1}{\sqrt{2}}\left(\psi_a(r_1)\,\psi_b(r_1) - \psi_b(r_1)\,\psi_a(r_1)\right)$$

$$= 0$$

Hence no probability of 2 particles having same position.
⇒ Pauli Exclusion Principle.

⊙ Do not confuse Antisymmetry & Parity.
Fermions are, by definition, antisymmetric under wavefunction exchange, which means that if you have 2 fermions, represented by states $\psi_1(x)$ and $\psi_2(y)$, then $\psi_1(x)\psi_2(y) = -\psi_2(x)\psi_1(y)$.
⊙ But odd Parity is something different. It applies to SINGLE particle and says that if you reverse all space coordinates then wave function switches sign. $\psi(x) = -\psi(-x)$

Models of Nucleus

Semi Empirical Mass Formula

To ~~measure~~ calculate/predict mass of nucleus w/o doing measurement.

$$M(Z, A) = ZM_p + (A-Z)M_n + \left(\frac{E_b}{c^2}\right)$$

do not forget the c^2

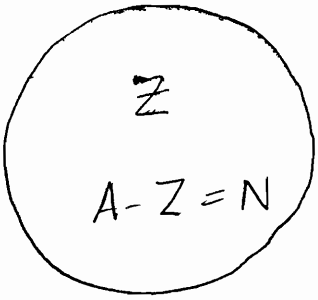
E_b is negative here
$E_b = -|E_b|$

We need to predict Binding Energy to predict nuclear mass. Hence its also called Semi Empirical Mass Formula
or
Semi Empirical Binding Energy Formula

Binding Energy is the energy by which nucleons are held together. Or the Energy required to break the nucleus into constituent elements.

Bohr said that nucleus is similar to liquid drop. So he gave Liquid Drop Model in 1936.
Density is independent of shape & size.

<u>Semi Empirical Mass Formula is correction upon this model.</u>



> **<u>Volume Energy</u>**
> Assume that nucleon-nucleon Bond Energy is $U \Rightarrow$ energy per nucleon is $\left(\frac{U}{2}\right)$
> Consider the nucleons to be packed having 12 adjoining nucleons, the energy per nucleon $= 12 \times \frac{U}{2} = 6U$
> As there are A nucleons in the nucleus
> Total binding Energy $= 6UA$
> $\Rightarrow E_v = a_1 A$ (say)
> E_v is called volume energy

According to this formula, E_b has 5 components

1. E_V volume energy
2. E_S Surface Energy
3. E_C Coulomb

} borrowre from liquid drop model of Bohr

4. $E_{Ass.}$ Asymmetry Energy * Asymmetry in no. of protons and neutrons.
5. E_P pairing Energy * {Even-Even} Pairs, {Odd-Odd} Pairs

① <u>Volume Energy</u> is <u>quantification of Strong Nuclear Force</u>

$E_v \propto$ Volume $\qquad E_v \propto -\frac{4}{3}\pi R_0^3$ (A) (attractive)

$$\boxed{E_V = -a_1 A}$$ attractive

Nucleons on surface are less stable as they have less neighbours. Hence can be removed more easily.

(2) <u>Surface Energy</u> $\propto +4\pi R_0^2 A^{2/3}$

$$\boxed{E_S = + a_2 A^{2/3}}$$ (repulsive)

It will adversely effect the stability of nucleus.

(3) <u>Coulumb Energy</u>

On an average, we can assume 1 proton at centre, 1 proton at radius

It is repulsive and hence it will negatively effect the stability of nucleus.

$$E_C = \frac{+e^2}{4\pi\varepsilon_0 R}$$

$$\text{Self Energy} = \frac{3}{5} \cdot \frac{Q^2}{4\pi\varepsilon_0 R} = \frac{+a_3 Z^2}{A^{1/3}} \qquad (R = R_0 A^{\frac{1}{3}})$$

$$\text{No. of Pairs} = {}^{Z}C_2 = \frac{Z(Z-1)}{2} \Rightarrow \boxed{E_C = \frac{+e^2 Z(Z-1)}{8\pi\varepsilon_0 R}}$$

$$\Downarrow$$

$$\boxed{E_C = \frac{a_3 Z(Z-1)}{A^{\frac{1}{3}}}}$$

$$\Rightarrow E_C \simeq \frac{+a_3 Z^2}{(A)^{1/3}}$$

$\Rightarrow$ By liquid drop model,

$$B \cdot E \cdot = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

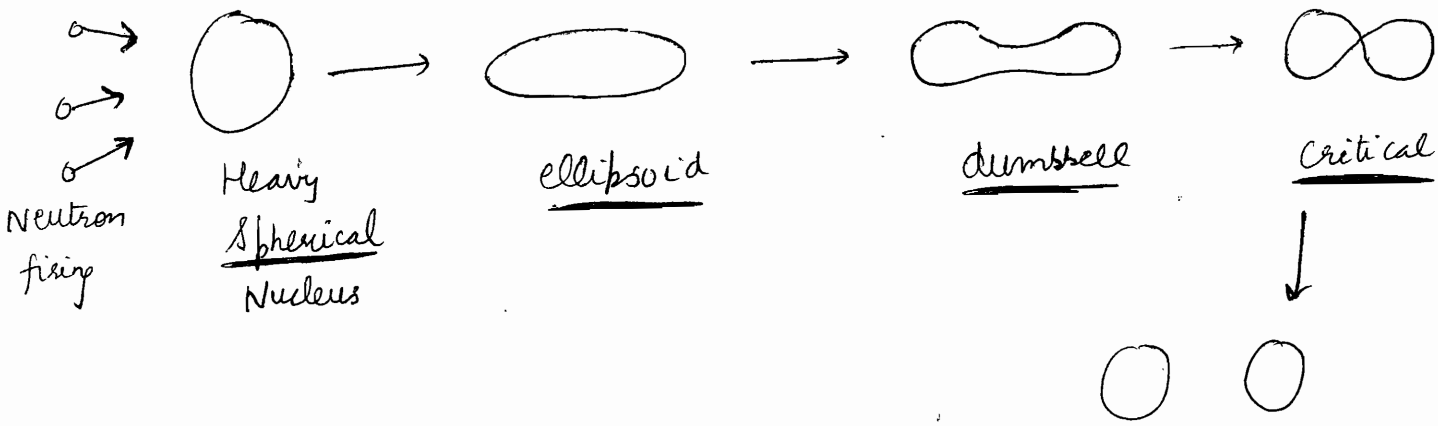
How to determine a_1, a_2, a_3 ??

We use semi-empirical techniques, similar to curve matching done in Einstein's C_V curve matching semi empirical approach.

Take any 3 representative nuclei. Calculate a_1, a_2, a_3 for them by using their Binding Energies (that are known beforehand)

But a lot of deviations are there when we use this model on other nucleus.

Fission is successfully explained by Liquid Drop Model.



This model was not discarded by Weizsacker & Bethe. They added more terms to this model.

4) Assymmetric Energy

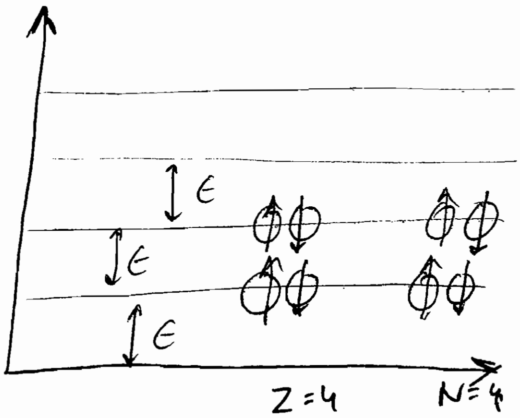
Energy corresponding to excess of neutrons / protons

Stability line upto Ca, N= Z line

Symmetry : $N = Z$

Assymmetry : $N > Z$ or $N < Z$

Assymmetry is due to excess of nucleons i.e. either $N > Z$ or $Z > N$.

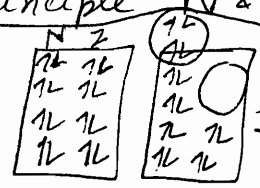


Nuclear Energy levels

$Z = N$: stable

$N + Z = 8$

Both are fermions. Both obey Pauli's Exclusion Principle N & [partial]



$\Delta \epsilon = \left(\frac{N-Z}{2}\right) \times \left(\frac{N-Z}{4}\right)$

In this case

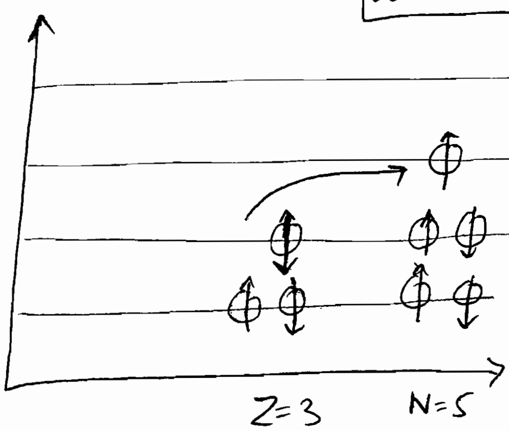
$\Delta \epsilon = \left(\frac{N-Z}{4}\right) * \varepsilon$

$\Rightarrow \Delta \varepsilon \propto \frac{(N-Z)^2}{A}$ as E [partial]

In fully filled states, nucleus is most stable

do this derivation from BT

Note that we are considering $A = \text{const}$



$N > Z$ or $Z > N$: unstable

$N + Z = 8$

$N - Z = 2$
↑
excess

Note that proton can be converted into [redacted] neutrons.

$$E_{assymmetry} = +\left(\frac{N-Z}{2}\right) \times \left(\frac{N-Z}{4}\right)\frac{\Delta E}{A}$$

↑ No. of shift

↑ Energy shift

↑ $\varepsilon \propto \frac{1}{A}$

$$E_{Ass} = + a_4 \frac{(N-Z)^2}{A}$$

↕

→ note that $(N-Z)^2 \Leftrightarrow (Z-N)^2 \Rightarrow E_{ass}$ is similar for any excess

Perfect

$$E_b = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A}$$

5) Pairing Energy

Statistical Prediction by using discrepancy in E_b and sum of $(E_V + E_S + E_C + E_{Asy})$

For odd 'A' nucleus : $E_p = 0$

For even 'A', even-even : $E_p = -a_5 A^{-3/4}$

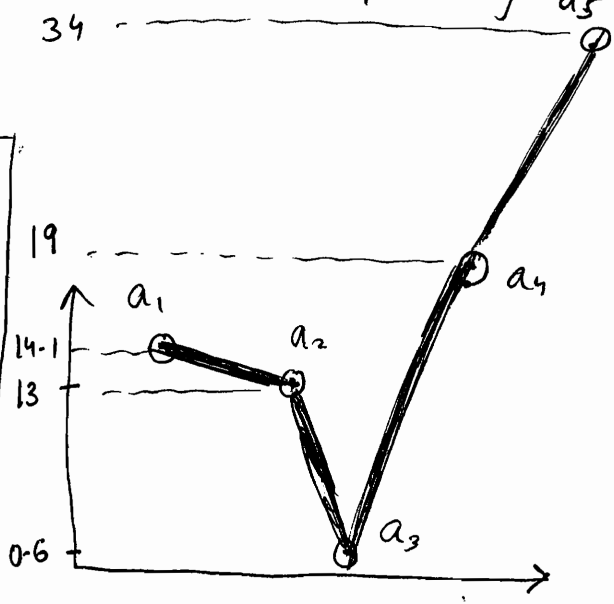
odd-odd : $E_p = +a_5 A^{-3/4}$

$$\boxed{E_p = (\pm, 0)\ a_5 A^{-3/4}}$$

a_1, a_2, a_3, a_4, a_5 are determined semi empirically by method of curve fitting.

a_1	=	14.1	MeV
a_2	=	13	MeV
a_3	=	0.6	MeV
a_4	=	19	MeV
a_5	=	34	MeV

Note this MeV



This model predicts Binding Energy and Mass of most of the nuclei.

[redacted]

Isobaric Series

[redacted]

$Z = 26$ to $Z = 30$ $A = 64$

Most stable :- Binding Energy Maximum Or Mass is minimum ISOBAR

$$\left.\frac{\partial E_B}{\partial Z}\right|_{Z = Z_0} = 0$$

$A = 64$

$$\frac{a_3}{A^{1/3}}(2Z_0 - 1) + a_4\left(-4 + \frac{8Z_0}{A}\right) = 0$$

$$\frac{0.6}{4}(2Z_0 - 1) + 19\left(-4 + \frac{8Z_0}{16}\right) = 0$$

Use "Z" so that no confusion between Z and 2

$$\Rightarrow 0.3Z_0 - \frac{0.6}{4} - 76 + \frac{38Z_0}{16} = 0$$

$(2.4 + 0.3)$ [redacted] $Z_0 = 76.15$

$Z_0 = \left(\frac{76.15}{\text{[redacted]}}\right)$ 2.7 $=$ [redacted] $= \underline{\underline{28.2}}$ ✓

take

$A = 27$

- (★) Note that Mass Formula calculates energy for <u>whole nucleus</u> and not per nucleon.

Mass Parabola

$$M(Z, A) = ZM_p + (A-Z)M_N + E_b$$

Note that we din't take c^2.... but its immaterial as
(i) $m_p \simeq m_n$ (Cassume)
(ii) we are interested in relations only

$$M(Z,A) = ZM_p + (A-Z)M_N + a_1 A + a_2 A^{2/3} + a_3 \frac{(Z^2 - Z)}{A^{1/3}} + a_4\left(A - 4Z + \frac{4Z^2}{A}\right) (\mp, 0) a_5 A^{-3/4}$$

★ USE ENERGY PARABOLA, RATHER THAN DIRECT DIFFERENTIATION AS
1. IT WILL FETCH MORE MARKS
2. ΔE calculation is much simpler

Now this equation is Parabola in Z. with A taken as const.

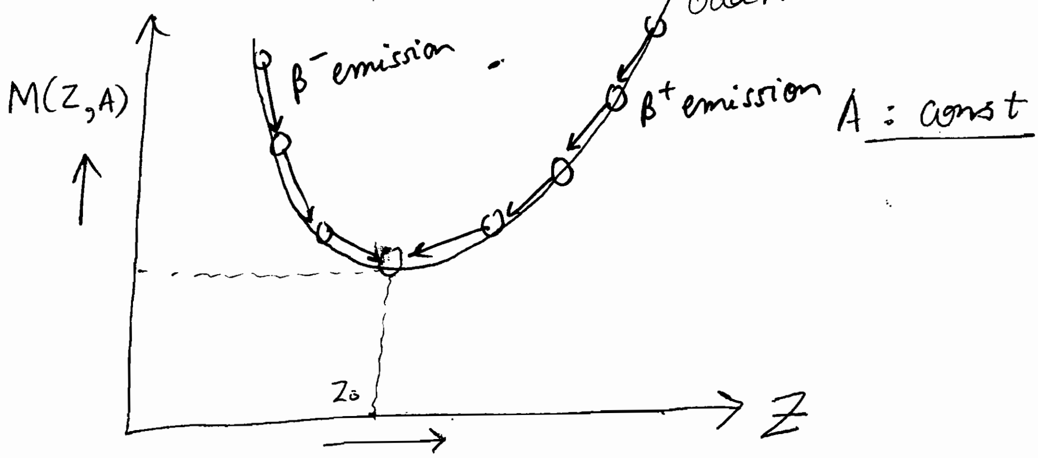
$$M(Z,A) = \left(M_N - a_1 + a_2 A^{-\frac{1}{3}} + a_4\right)A + \left(M_p - M_N - \frac{a_3}{A^{1/3}} - 4a_4\right)Z + \left(\frac{a_3}{A^{\frac{1}{3}}} + \frac{4a_4}{A}\right)Z^2 (0, \pm) E_p$$

$$\Rightarrow \boxed{M(Z,A) = \alpha A + \beta Z + \gamma Z^2 (\mp, 0) E_p}$$

For odd A nucleus (odd - even, even - odd)

$$M(Z,A) = \alpha A + \beta Z + \gamma Z^2$$

Mass Parabola



Note that Mass Parabola is valid for any isobaric series. Whatever is lying on vertex is most stable isobar.

For $Mass_{minimum} \Rightarrow \left(\frac{\partial M}{\partial z}\right)\Big|_{z=z_0} = 0$

Minimum Mass @ $\boxed{Z_0 = -\left(\frac{\beta}{2\gamma}\right)}$ $= \dfrac{\left(\frac{a_3}{A^{1/3}} + 4a_4\right)}{2\left(\frac{a_3}{A^{1/3}} + \frac{4a_4}{A}\right)}$

$A \doteq 64 \Rightarrow Z_0 = -\frac{\beta}{2\gamma} = 28.4 \approx \underline{28}$ (assume $M_P \approx M_N$)

[redacted]

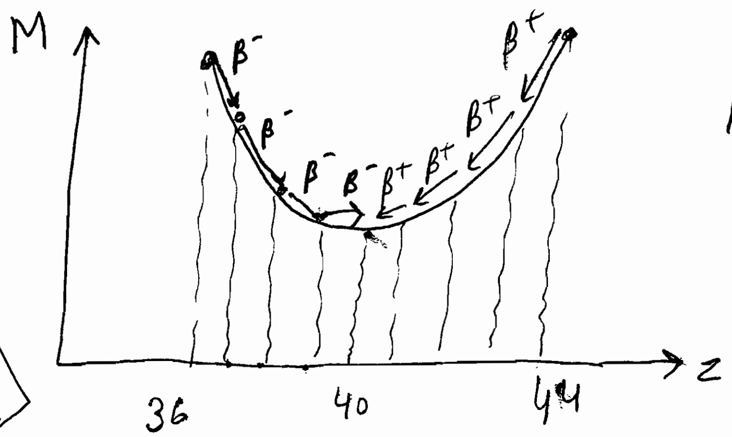
<u>It also predicts stability against $\beta^{\pm}$ decay. Any isobaric series try to converge at Z_0 via $\beta^{\pm}$-emission</u>

eg. 91 series

$^{91}_{36}Kr$ .... for this series : most stable element is 40.

Note that in β decay, mass number does not undergo any change

For A = 91, $Z_0 = -\frac{\beta}{2\gamma} = 39.3 \approx \underline{40}$



A = 91

This parabola also gives energy given out $[Z \to Z_0]$

$$\Delta E = M(Z_0, A) - M(Z, A)$$

$$= \beta(Z_0 - Z) + \gamma(Z_0^2 - Z^2)$$

(Replace β using Z_0 expression)

$$= -2\gamma Z_0^2 + \gamma Z_0^2 + 2\gamma Z_0 Z - \gamma Z^2$$

$Z_0 = -\left(\frac{\beta}{2\gamma}\right)$

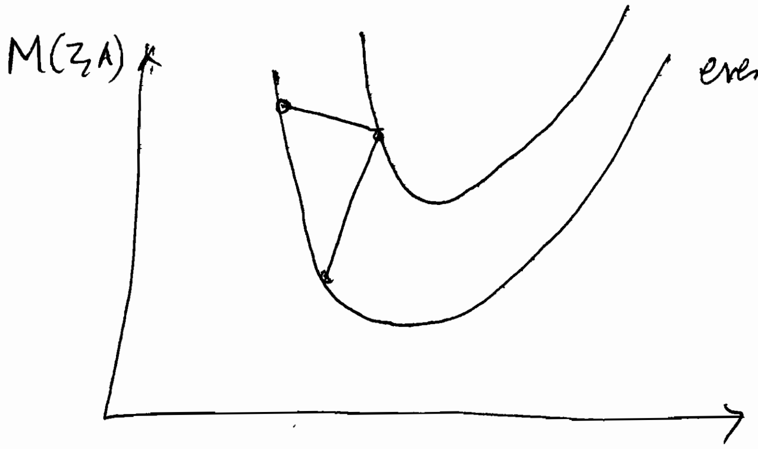
$$\boxed{\Delta E = -\gamma(Z - Z_0)^2}$$

$\gamma = \left[\frac{a_3}{A^{1/3}} + \frac{4a_4}{A}\right]$

(*) Mass Parabola, therefore, gives :

1. Stability against β decay
2. Most stable isobar
3. ΔE in transition

For even A nucleus, [odd-odd / even-even]



For even nuclei

$n \to p$

$n \to n-1$ odd → even
$p \to p+1$ odd → even
(odd-odd) to (even-even)

while total A is conserved.!!

[redacted]

1. B.E. or mass of nucleus
2. Most stable element
3. size of nucleus & coulomb repulsion term can be obtained.
4. Nature of Force can be predicted.

Size of Nucleus

Take 2 mirror nuclei

$$M_A(Z,A) = ZM_p + (A-Z)M_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{1/3}} + \underbrace{a_4 \frac{(A-2Z)^2}{A}}_{= \frac{(a_4)}{A}}$$

For a mirror nucleus,

$N - Z \sim 1$

$A - 2Z \sim 1$ & N and Z are interchanged

∘ They will then be odd - even ⇒ no factor of a_5

For other nuclei

$$M_B(Z,A) = (A-Z)\,M_p + Z M_n - a_1 A + a_2 A^{2/3} + a_3 \frac{(A-Z)(A-Z-1)}{A^{1/3}} + a_4 \underbrace{\frac{(A-2Z)^2}{A}}_{= \frac{a_4}{A}}$$

Difference of Mass of 2 nucleus
$\simeq$ Difference of Binding energy of 2 nucleus:

$$= \boxed{E_b - E_b' = a_3 \frac{Z(Z-1)}{A^{1/3}} - a_3 \frac{(A-Z)(A-Z-1)}{A^{1/3}}}$$

[Hence by mirror-nuclei method, a_3 can be predicted.
Hence R_0 is predicted]

$\Rightarrow$ $R = R_0 A^{1/3}$: radius of nucleus is known.

Yukawa Potential

(35)

$$\boxed{\phi(r) = -\left(\frac{g}{4\pi\varepsilon}\right) e^{-(r/r_0)}} \qquad r_0 = \left(\frac{\hbar}{mc}\right)$$

★ $E_c = Z C_2 \cdot \dfrac{e^2}{4\pi\varepsilon_0 r}$ $\qquad r = r_0 A^{1/3}$

$$= \frac{Z(Z-1)}{A^{1/3}} \left(\frac{e^2}{8\pi\varepsilon_0 r_0}\right) \qquad \Rightarrow \boxed{a_3 \propto \left(\frac{1}{r_0}\right)}$$

$$\Rightarrow \boxed{a_3 = \frac{e^2}{8\pi\varepsilon_0 r_0}} \quad ✓$$

Particle Physics (2)

⊛ **Isotopic Spin** "I" is fictitious spin.

"I" represents <u>family of elementary particles</u> having all properties same except charge.

It's a vector quantity. 'Spin' in name is misnomer.

✱ { <u>I_3 is the component of isotopic spin along z-axis.</u>
<u>It represents member in the family. (i.e. charge state)</u> }

I_3 ~~varies~~ varies $-I$ to $+I$ ($2I+1$ values)

<u>$(2I+1)$: no. of observed charged states of an elementary particle.</u>

I is defined only for Hadrons, i.e. particles taking part in Strong & EM field interactions.

For Nucleons $2I+1 = 2$ [no. of states = 0 and 1)

0 ↑ neutron, 1 ↑ proton

$\Rightarrow I = \left(\frac{1}{2}\right)$

$\Rightarrow$ for ~~nucleons~~ : $I = \left(\frac{1}{2}\right)$

$I_3 = -\frac{1}{2}$ for neutron
$I_3 = +\frac{1}{2}$ for proton

⇐ Note that we are treating 'n' and 'p' as 2 charge states of nucleon family !!

π Meson

$2I+1 = 3$

$\Rightarrow I = 1$

$[\pi^{\pm}, \pi^{0}]$

$\Rightarrow I = 1$ for all Mesons

$I_3 = -1$	for	π^{-}
$I_3 = 0$	for	π^{0}
$I_3 = +1$	for	π^{+}

Antiparticles have Opposite Isotopin Spin...

K Meson

$2I+1 = 2$

$I = \left(\frac{1}{2}\right)$

$I_3 = \frac{-1}{2}$	for	k^{0}
$I_3 = \frac{+1}{2}$	for	k^{+}

η Meson

$2I+1 = 1 \Rightarrow I = 0$

$I_3 = 0$ for η^{0}

Ξ Hyperon

$2I+1 = 2$

$I = \left(\frac{1}{2}\right)$

Ξ^{0}	$+\left(\frac{1}{2}\right)$
Ξ^{-}	$-\left(\frac{1}{2}\right)$

Λ^{0} Hyperons

$2I+1 = 1$

$\Rightarrow I = 0$

$I_3 = 0$ for Λ^{0} hyperons

$I_3 = 0$ for Ω^{-} hyperon

Σ Hyperons

Σ^{+}	$+1$
Σ^{0}	0
Σ^{-}	-1

→ I_3 is conserved in strong as well as EM interactions.

→ I is conserved in only strong interactions

इसें बस I_s से ही मतलब है!!

Strangeness Number

$$S = 2(Q - I_3) - B$$

i.e. $$Q = I_3 + \left(\frac{S+B}{2}\right)$$ ✓

Gell-Mann Nishijima Formula

This is magnitude of charge. Charge = Qe

$$Q = I_3 + \frac{Y}{2}$$

I_3: Isotopic Spin Component, Y: hypercharge = $S+B$

For π mesons :

Illustration of Gell-mann Nishijima Formula :-

π⁺ Meson

$$S = 2(1-1) - B = 2(0) - 0$$
$$= 0$$

⇒ π^+ Meson is non strange particle

π^- Meson

$$S = 2(-1+1) - 0 = 0$$

π^0 Meson

$$S = 2(0-0) - 0 = 0$$

⇒ π mesons are non strange particles

For η^0 Meson

$$S = 2(0-0) - 0 = 0$$

For neutron

$$S = 2\left(0+\frac{1}{2}\right) - 1 = 0$$

For Proton

$$S = 2\left(1-\frac{1}{2}\right) - 1 = 0$$

$S = 0$: Non strange : $\pi^{\pm}, \pi^{0}, n, p, \bar{n}, \bar{p}, \eta^{0}$

$S = 1$: Strange : $\bar{\Sigma}^{-}, \bar{\Sigma}^{+}, \bar{\Sigma}^{0}, \bar{\lambda}^{0}, k^{0}, k^{+}$

All hyperons are strange $\longrightarrow$ $S = -1$ ($\lambda^{0}, \Sigma^{\pm}, \Sigma^{0}$), $S = -2$ (Ξ^{0} ...), $S = -3$ (Ω^{-})

k-Mesons $\longrightarrow$ $S = +1$

$S = 2$: $\bar{\Xi}^{0}$ $\Xi^{(+)}$

$S = 3$: Ω^{+}

Antiparticles have Opposite strangeness number.

k, λ, Σ, Ξ, Ω — Strange

$S = -1$: $\lambda^{0}, \Sigma^{+}, \Sigma^{-}, \Sigma^{0}$

$S = -2$: $\Xi^{(0)}, \Xi^{(-)}$ $\longrightarrow$ They are produced by Strong Interaction & decaying is via weak interaction.

$S = -3$: Ω^{-}

$\longrightarrow$ They are produced as pair production

Hypercharge

$$Y = S + B$$

-3	-2	Strangeness -1		1	2
Ω^{-} $2(-1-0)-1$ $= -3$	$\Xi^{(0)}$ $2(0-\frac{1}{2})-1$ $= -2$	λ^{0} $2(0-0)-1$ $= -1$	Σ^{0} $2(0-0)-1$ $= -1$	k^{0} $2(0+\frac{1}{2})-0$ $= +1$	None
	$\Xi^{(-)}$ $2(-1+\frac{1}{2})-1$ $= -2$	$\Sigma^{(+)}$ $2(1-1)-1$ $= -1$		$k^{(+)}$ $2(1-\frac{1}{2})-0$ $= +1$	Note that antiparticle has $S = -S_{Particle}$
		$\Sigma^{(-)}$ $2(-1+1)-1 = -1$			

Tut

Leptons take part in Weak Interaction & Cons. laws
Baryons take part in strong & EM interactions. They
follow 17 Conservation laws.

Q/ leptons in EM interaction? Of course since they have Charge & mass
⇒ EM & gravitation reaction तो होंगे ही !!

(★) Mesons have 0 spin, 0 lepton number, 0 baryon number. Therefore, can be easily fit into any reaction.

(9) (i) $\mu^- \longrightarrow e^- + \bar{\nu}_e$

linear momentum and mass energy not checked !!

	μ^-	e^-	$\bar{\nu}_e$	
Q	-1	-1	0	$\Delta Q = 0$
S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	Not Conserved
L_e	0	1	-1	$\Delta L_e = 0$
L_μ	1	0	0	Not Conserved
L_τ	0	0	0	$\Delta L_\tau = 0$
B	0	0	0	$\Delta B = 0$

* Hence not allowed due to non conservation of S and L_μ

(ii) $n \Rightarrow p + e^-$

	n	p	e^-
Q	0	1	-1
✓ S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
✓ L_e	0	0	1

$\Delta Q = 0$

Not Conserved

$\Delta L_e \neq 0$ not conserved

Neutron cannot decay into proton & electron

(iii) $k^\circ \rightarrow \pi^\circ + \pi^\circ$

⊙ leptons are not involved
⊙ no Baryons involved

S : 0   0   0

Q : 0   0   0

$\Delta S = 0$

⊙ All Mesons have 0 spin

Allowed

Strangeness need not be conserved for weak interaction eg. decay process

Strangeness   +1   0 0 Δ Strangeness $\neq 0$

⇓

✓ ⇒ decay reaction goes by weak interaction

(iv) $e^+ + e^- \longrightarrow \gamma + \gamma$

	e^+	e^-	$\gamma+\gamma$	
L_e	~~+1~~ -1	+1	0	$\Delta L_e = 0$
S	$(\frac{1}{2})$	$(\frac{1}{2})$	1	$\Delta S = 0$

(11)

(i) $n \rightarrow p + e^- + \nu_e$

	n	p	e^-	ν_e	
L_e	0	0	1	+1	$\Delta L_e \neq 0$
B	1	1	0	0	$\Delta B = 0$
S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\Delta S = 0$

(*) $S = \frac{3}{2}$ $S = \frac{1}{2}$ are same ✓

✓ $n \rightarrow p + e^- + \overline{\nu_e}$ ⇐ allowed reaction

$p \rightarrow \pi^0 + e^- + \nu_e$

	p	π^0	e^-	ν_e	
Q	1	0	-1	0	$\Delta Q \neq 0$
ΔL_e	0	0	+1	1	$\Delta L_e \neq 0$
B	1	0	0	0	$\Delta B \neq 0$

$p \rightarrow \pi^0 + e^+ + \nu_e$

	p	π^0	e^+	ν_e	
$\Delta L_e = 0$					$\Delta B \neq 0$
B	1	0	0	0	
S	$(1/2)$	0	$(1/2)$	$(1/2)$	

	$n \rightarrow$	p +	e^- +	γ
S	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
L_e	0	0	1	0

Photon is a boson

✓ $\Delta S \neq 0$

✓ $\Delta L_e \neq 0$

✓ $n \rightarrow p + e^- + \bar{\nu}_e$ — β decay (β^-)

✓ $p \rightarrow n + e^+ + \nu_e$ — β decay (β^+)

✓ $p + e^- \longrightarrow n + \nu_e$ — β decay (electron capture)

$\pi^- + p \rightarrow k^+ + k^-$ ✗

Pair Production Reaction

Q	-1	+1	+1	-1	$\Delta Q = 0$
S	0	$\frac{1}{2}$	0	0	Not Conserved
B	0	1	0	0	Not Conserved

$\pi^- + p \rightarrow \Sigma^+ + k^-$ ✓

	π^-	p	Σ^+	k^-	
Q	-1	+1	+1	-1	$\Delta Q = 0$
B	0	+1	+1	0	$\Delta B = 0$
S	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\Delta S = 0$

Strangeness $\pi^- + p \longrightarrow \Sigma^+ + k^-$

$0 \quad 0 \quad -1 \quad +1$ $\boxed{\Delta S = 0}$

$\Downarrow$

either EM or strong

$I_3 \quad -1 \quad +\frac{1}{2} \quad 1 \quad -\frac{1}{2}$: Not Conserved

$\Rightarrow$

Check @ Home

---

(18) $\Sigma^0 \longrightarrow \lambda^0 + \gamma$ (lifetime $\leq 10^{-14}$ s)

$\Downarrow$

EM

(Q) $0 \quad 0 \quad 0$ conserved ✓

(Sp) $\frac{1}{2} \quad \frac{1}{2} \quad 1$ Conserved ✓

(B) $1 \quad 1 \quad 0$ Conserved ✓

$\Delta S = 0$

$\Downarrow$

reaction proceeds via EM or strong

(I_3) $0 \quad 0 \quad 0$ Conserved

Now lifetime $\leq 10^{-14}$ s and γ photon involved

$\Rightarrow$ EM interaction

Strange Particle < production : conserved } Strangeness Quantum No.
decay : not conserved

Strange Particle Production & Decay

Q-13 Tut Sheet
अरे भ्रष्टा, यहाँ 4 हैं !!

[redacted]

① $\pi^- + p \rightarrow \lambda^0 + k^0$

$\pi^+ + n \rightarrow \lambda^0 + k^+$

⇒ Strong/EM reaction
⇓
check I, Str. after checking 8 conservation rules.

② $k^- + p \rightarrow \Omega^- + k^+ + k^0$
(antiparticle)

③ $p + p \rightarrow p + \lambda^0 + k^+$

⊙ Prodⁿ by strong force ⊛
⇒ strangeness conserved
⇒ λ^0 ke साथ k^0/k^+ तो आयेगा

⊙

[redacted]

① $\Omega^- \rightarrow \Sigma^0 + \pi^-$

	Ω^-	Σ^0	π^-	
I_3:	0	0	-1	
I	0	1	1	: integral

② $\Sigma^+ \rightarrow p + \pi^0$
$\rightarrow n + \pi^+$

I_3: +1 ; $(\frac{1}{2})$; (1)

③ $\Xi^- \rightarrow \lambda^0 + \pi^-$

④ $\Xi^0 \rightarrow \lambda^0 + \pi^0$

⇒ weak interaction
All 8 conserved, then 'I_3' and 'strangeness' not conserved
Also Parity not conserved
Note that Parity is multiplicative.... rest are additive

⑤ $\lambda^0 \rightarrow p + \pi^-$ ⊛

$k^0 \rightarrow \pi^+ + \pi^-$ ⊛

बाकी (due to strangeness) λ^0 की necessity के लिए nucleon

⊛ k^+/k^0 का mass π के बराबर है ∴ production or decay from π particles only
k^0 from/to π^-
k^+ from/to π^+

* 8 Conservation laws ⇒ conserved ⇒ allowed

Now check, I_3, Strange, Parity : ⇒ EM/ Strong (if conserved) or Weak (if not conserved)

Quark Structure of Hadrons

✓ These are very heavy particles that take part in Strong Interactions.

✓ lifetime < 10^{-26} s : Product of Strong Interactions

← Resonant Particles.

They are not written in table

✓ Internal Structure of Hadrons in terms of 6 quarks.

None of the quarks have been discovered.
Its only a model.

→ Due to phenomenon called "Colour Confinement", quarks are never observed or found in isolation, they can only be found within baryons or mesons.

Therefore, much of what is known about quarks, has been drawn from observation of hadron themselves. Quarks, by default, clump together to form groups, or hadrons. The 2 types of hadrons are Mesons (1 quark, 1 antiquark) and Baryons (3 quarks). The constituent quarks in a group cannot be separated from their parent hadron & that is why quarks can never be studied at more direct level than at hadron level.

Nuclear Physics (3)

3/03/12

Magic Numbers

Z=20 : Ca, Z=28 : Fe, Z=50 : Tin, Z=82 : Pb

2, 8, 20, 28, 50, 82, 126

Number of neutrons OR protons, whose nuclei are most stable.

eg. $^{4}_{2}He$: doubly magic nuclei P and N ; both are of magic no. 2

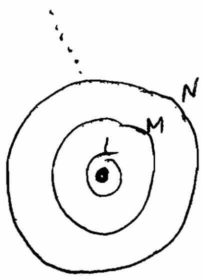
Shell Model is an attempt to explain stability of such nuclei.

$^{16}_{8}O$

20 : Calcium

28 : Iron

82 : lead



K : $n=1$ 2e max

L : $n=2$ 8e- max

M : $n=3$ 18e- max

⋮

$2n^2$

We know, Closed shell provide stability to atom

similarly, Nuclei with magic number of proton / neutron are most stable.

Q What is the [redacted] ??

→ 2 types of motion :

space motion (r, θ, ϕ)

Spin (s)

Similar to H atom problem, we can solve for other atoms. We will get quantum numbers for all these parameters

$$\begin{array}{cccc} (r, & \theta, & \phi) & (s) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ n & l & m_l & m_s \end{array}$$

n : unrestricted

l : $0, 1, 2, \ldots (n-1)$

m_l : $-l$ to l

m_s : $\pm\left(\frac{1}{2}\right)$

⇒ Total no. of states of e^- in a given n

$$= 2\sum_{l=0}^{n-1} 2l+1$$

$$= 2\left[2\frac{(n-1)(n)}{2} + (n)\right]$$

$$= 2\left[n^2 - n + n\right]$$

$$= 2n^2$$

→ Similarly, we can derive magic number reasoning in shell model.

Shell Model

[redacted]

① We have to consider motion of 1 nucleon due to Potential of (A-1) nucleons. We can assume it Yukawa's Potential or any other potential. Spherically symmetrical

② Every nucleon will possess n, l and m_l. We are interested in l and m_l.

$$L = \sqrt{l(l+1)}\,\hbar \qquad L_z = m_l \hbar$$

Similar to e^-, even for nucleons,

l	=	0	1	2	 3	4	5	6
		↓	↓	↓	↓	↓	↓	↓
		S	P	D	... F.	G	H	I

③ Each nucleon will have Spin [redacted] Angular Momentum.

$$|\vec{S}| = \sqrt{s(s+1)}\,\hbar \qquad S_z = m_s \hbar$$

④ Total Angular Momentum (Nuclear Spin) will be Obtained by L-S coupling or I-I coupling.

$$\vec{I} = \vec{L} + \vec{S} \qquad \vec{I} = \vec{I_1} + \vec{I_2}$$

$$\vec{L} = \Sigma \vec{l_i} \qquad \vec{I_i} = [redacted] (\vec{l_i} + \vec{s_i})$$

$$\vec{S} = \Sigma s$$

⑤ In 1 level of I, (2I+1) states

⑥ Nucleons form closed sub-shells within the nucleus in a similar manner as the e- do in the case of atoms. Hence, its named shell model.

SPUDS if PUG

DISH of PIG

Sp~~p~~ds ~~of~~ p~~u~~g d~~i~~sh of pig

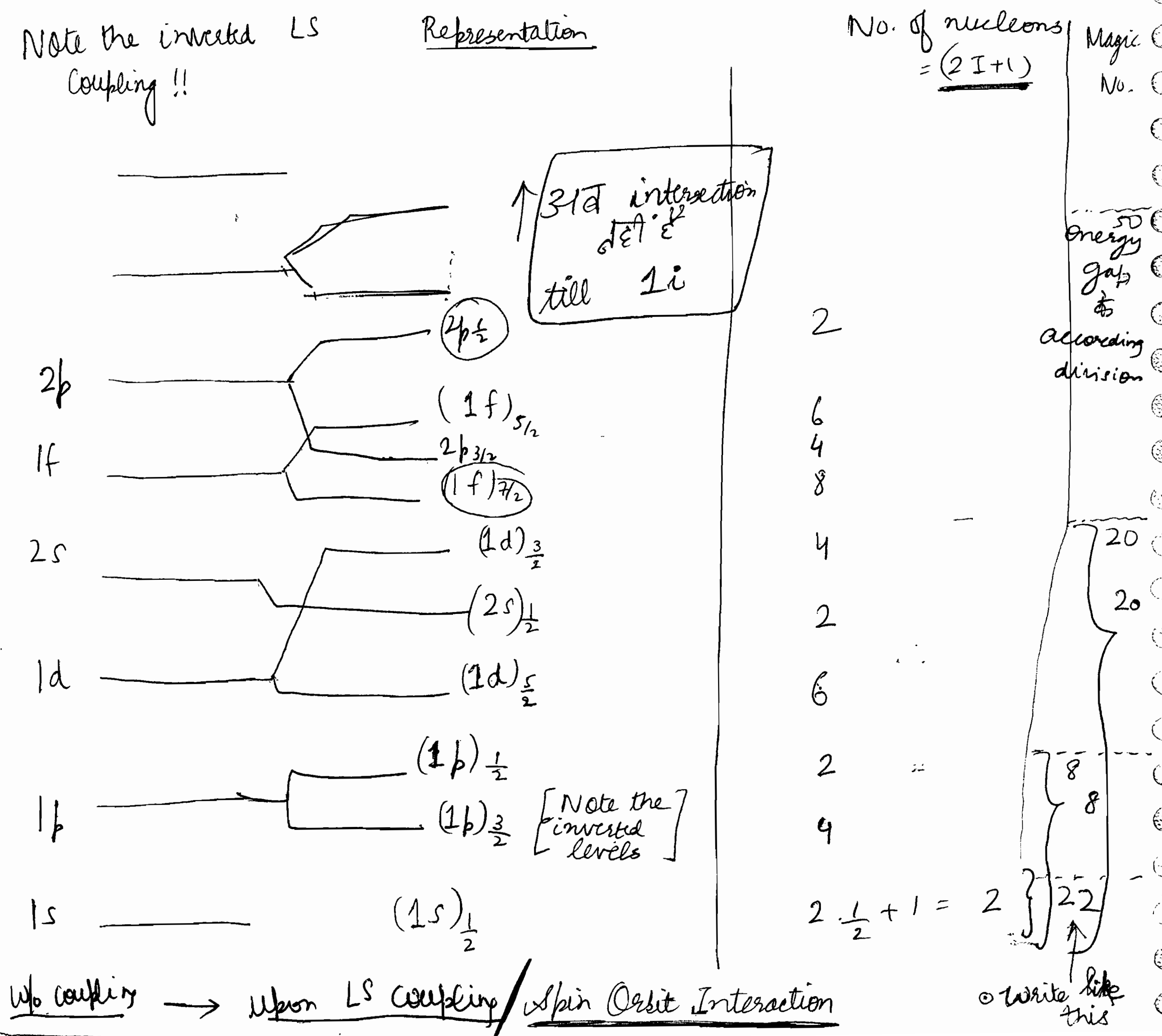
deletion of all vowels except the last gives us the order,

1s 1p 1d 2s 1f 2p 1g 2d 3s 1h 2f

3p 1i 2g . . .

Representation $(nl)_I$ $\boxed{I = l + s}$

Note the inverted LS Coupling !!

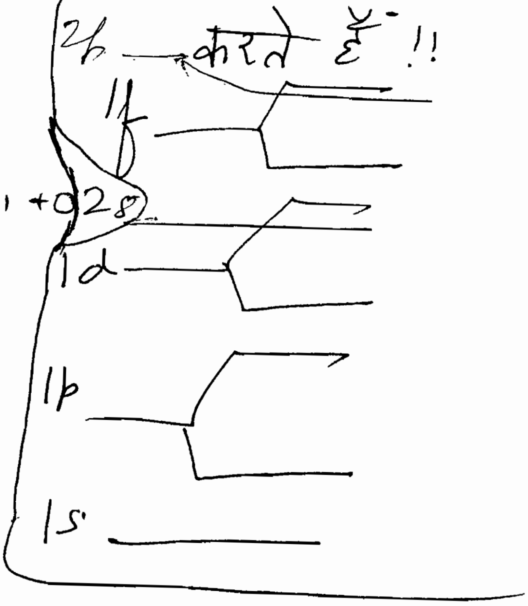


$A = 13$

$\left(1s_{\frac{1}{2}}\right)^{2+2}$ $\left(1p_{\frac{3}{2}}\right)^{4+4}$ $\left(1p_{\frac{1}{2}}\right)^{1}$

इसमें केवल 2p तक draw करता है !!

2s and 2p के फालतू में तंग करते हैं !!



eg. $^{13}_{6}C$

$\left(1s_{\frac{1}{2}}\right)^{2+2}$ $\left(1p_{\frac{3}{2}}\right)^{4+4}$ $\left(1p_{\frac{1}{2}}\right)^{1+0}$

fully filled nucleons contribute nothing to energy

$$\text{Parity} = (-1)^{l}$$ multiplication

✓ For even nucleons, $I = 0$, Parity is even — Case 1 — Odd A, i.e. odd-even or even-odd

So we will write $I = \left(\frac{1}{2}\right)^{-}$ for $^{13}_{6}C$

(spin I; Parity)

[redacted]

✓ It predicts ground states & excited states of all nuclei correctly. ✓ It predicts spin & parity of all nuclei correctly.

Parity

For odd-odd nucleons: $(-1)^{Zl_1} (-1)^{Nl_2} = (-1)^{(Zl_1 + Nl_2)}$ for all types

For even-even: even parity — Case 2 — even A (even-even)

$\vec{I} = \vec{I}_p + \vec{I}_N$

Case 3 even A

odd-odd

spin

$I = I_p \pm I_N$ depending on odd/even ness of

$|I_1 + I_2 + l_1 + l_2|$

If $|I_1 + I_2 + l_1 + l_2|$ is even $\Rightarrow I = I_p - I_N$

If $|I_1 + I_2 + l_1 + l_2|$ is odd $\Rightarrow I = I_p + I_N$

Parity multiplicative

Nordheim's Strong Rule

2 more successes — Total 6 success

(3) It predicts magnetic moment of few nuclei correctly.

(4) It predicts Quadrupole Moment of few nuclei correctly.

$$\vec{\mu} = \frac{e}{2m_p} g \vec{I}$$

$$Q = \left(\frac{3}{5}\right)\left(\frac{2I-1}{2I+2}\right) R_0^2 A^{2/3} \text{ Barns}$$

$r^2 = R_0^2 A^{\frac{2}{3}}$

For nuclei far off from magic number, their $\vec{\mu}$ and Q are not predicted.

(5) It successfully explains Nuclear Isomerism.

<u>Nuclear Isomers are long lived excited state of nuclei.</u>

(A,Z) are same for isomers but states are different.

X $\quad X^*$: isomer of X. Its lifetime is higher.

long lived excited states arise when Probability of Transition is low, i.e. when it violates selection rules of transition

eg $1^1 f_{\frac{5}{2}} \longrightarrow 2^1 p_{(\frac{3}{2})}$ $\quad \underline{\Delta l = \pm 1}$ is violated

(6) It explains extra-stability enjoyed by magic numbers. Hence elements ~~components~~ near magic no. have many isobars & isotopes.

[redacted]

(1) 4 stable nuclei do not fall into magic numbers.

[illegible] B

$^2_1H \quad ^6_3Li \quad ^{10}_5B \quad ^{14}_7N$ (odd-odd)

(2) Few orbitals are overlapping eg. 2s and 1d

This is not in accordance with shell model.

(3) It does not predict spin of $^{23}_{11}Na$ nucleus correctly.

$(1s)^2 \quad (^1p)^6 \quad (^1d_{\frac{5}{2}})^4$ [redacted] even : 0 : Neutrons

$(1s_{\frac{1}{2}})^2 \quad (1p_{\frac{3}{2}})^4 \quad (1p_{\frac{1}{2}})^2 \quad (1d_{\frac{5}{2}})^3$ odd $\Rightarrow (\frac{5}{2})^+$: Protons

~~Parity~~ ~~is~~ Predicted : $(\frac{5}{2})$

Observed : $I = \frac{3}{2}$

(4) It does not explain Magnetic Moment & Quadrupole Moment of numbers away from magic numbers.

(4)

$^{13}_{7}N$

Only due to Z

$(1s_{\frac{1}{2}})^2 \ (1p_{\frac{3}{2}})^4 \ (1p_{\frac{1}{2}})^1$

Parity: odd

$I = \left(\frac{1}{2}\right)^-$

$^{17}_{8}O$

Only due to N

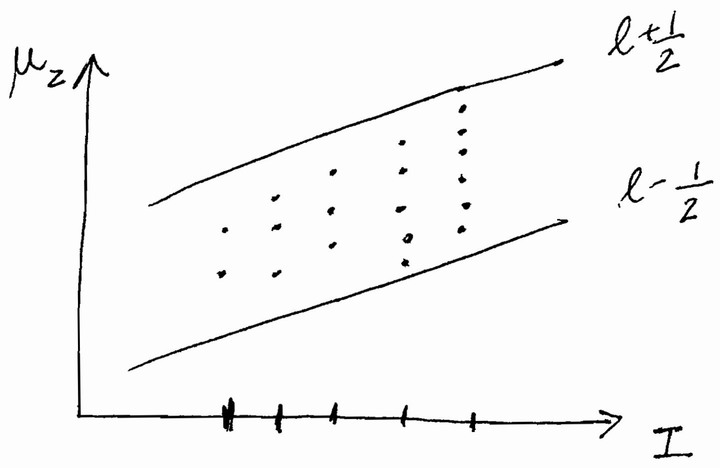
$N = 9$

$(1s_{\frac{1}{2}})^2 \ (1p_{\frac{3}{2}})^4 \ (1p_{\frac{1}{2}})^2 (1d_{\frac{5}{2}})^1$

$I = \left(\frac{5}{2}\right)^+$

(*) μ_z not predicted due to

1. Meson distortion in internal structure of nucleons
2. Spherically symmetrical Potential assumed for deriving shell model.



Another Points about shell model

- The neutron levels are equally spaced whereas the spacing between the proton levels increases gradually as we move from the ground level to the higher level. This is due to Collumb's force or repulsion of protons.

In adding nucleons to the nucleus, the proton and the neutron levels should be equally filled up for maximum stability. Hence for small values of mass number A, number of neutrons is very nearly equal to number of protons. As the no. of nucleons becomes large, the most stable nuclides will again be formed when lowest energy levels available to protons and neutrons are filled up. As the spacing between proton levels goes on increasing, no. of neutrons will be greater than no. of protons for stable nuclei. Hence another success for shell model.

Evidences for shell model

1. Nucleons have tendency to form pairs. It is easier to remove a single unpaired nucleon from the nucleus than the one which is paired.
2. Nuclei containing even no. of protons and neutrons are most abundant; nuclei with both p and n odd are least.
3. High stability for magic numbers.
4. Nuclides whose Z correspond to magic number have highest no. of isotopes. eg. $_{50}Sn$ has 10 isotopes.
5. Stable end-product of natural radioactive series $^{208}_{82}Pb$. Z = 82, N = 126 : both magic numbers.
6. 1st excitation energy of magic numbers of nucleon in a nuclei is very high. When the no. of protons or neutrons exceed magic number by one, binding energy of extra nucleon is much less than avg. value.
7. Island of isomerism for $19 < Z < 37$ due to transition from 2p to 1f ~~shell~~ subshells which is against selection rule.

Lecture 2
Appendix

Elementary Particles

- ½ integral spin → **Fermions**
  - [all are dirac fermions i.e. have their distinct antiparticle]
  - [They are basic building block of all matter]
  - interact via strong force → **Quarks**
    - [fundamental constituent of hadrons and interact via strong force]
    - up-type ⊕ve → up, charm, top
    - down-type ⊖ve → down, strange, bottom
  - interact via weak force → **Leptons**
    - e⁻ like ⊖ve → e^-, μ^-, τ^-
    - neutrinos (neutral) → ν_e, ν_μ, ν_τ
- integral spin → **Bosons**
  - [Fundamental forces of nature are mediated by gauge bosons.]
  - [Mass is hypothesized to be created by Higgs Boson]
  - [Graviton is added to the list, though not predicted]
  - Graviton
  - Higgs Boson
  - Gauge Boson
    - Photon
    - $W^{\pm}$ Z^0 boson ← intermediate vector bosons
    - Gluons

Quark	u	c	t	γ	
	d	s	b	g	
	ν_e	ν_μ	ν_τ	Z^0	Gauge Bosons
Leptons	e	μ	τ	$W^\pm$	

Standard Model (elementary particles)

(*) eithere be in upper CUT वरना नीचे तो BHASAD hai

⊙ all exchange particles are bosons

[redacted]

[defined as strongly interacting composite particles..... made up of quarks]

- **fermions** → **Baryons** (3 quark) — odd no. of quarks, $S = \frac{1}{2}, \frac{3}{2}$
  - **Nucleons**
    - P
    - N
  - 1 or more strange quark → **Hyperons**
    - Λ^0
    - Σ ($\Sigma^+, \Sigma^0, \Sigma^-$)
    - Ξ (Ξ^0 & Ξ^-)
    - Ω (Ω^-)
- **bosons** → **Mesons** (1 quark, 1 antiquark) — even no. of quarks, $S = 0$
  - [They mediate residual strong force between nucleons]
  - Pion: π^+, π^0, π^-
  - Eta Meson: η
  - Kaon: K^+, K^0

[redacted]

Smallest neutral particles into which matter can be divided by chemical reactions

[redacted]

Smallest particles into which a non-elemental substance can be divided while maintaining PHYSICAL properties of substance.

Particle [redacted] Physics (3)

05/03/2012

Conservation laws (contd...)

→ Process involving photon proceeds by EM interaction. Also I_z is not conserved

Charge Conjugation

Replace particle by antiparticle. If reaction proceeds ⇒ charge conjugation is conserved.

Parity

$$A + B \longrightarrow C + D$$

if $P(A) * P(B) = P(C) * P(D)$ ⇒ Parity conserved

Also, $P = (-1)^l$

[redacted]

Replace t by $-t$.

looking at movie backwards & we are not able to differentiate b/w $f(t)$ and $f(-t)$

⇒ Time Reversal Conservation

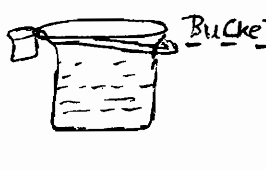
→ CPT is supposed to be conserved in all nuclear reactions.

Gillmann Nishijima Relationship (modified)

$$Q = \left[I_z + \frac{1}{2}(S + B + c + t + b) \right]$$

(c ↑ Charm, t ↑ top, b ↑ bottom)

$q = Qe^-$



Quark Model of Hadrons

In 1964, Gellmann and Neemann proposed that hadrons are massive particle who can further be written as combination of 'aces', later on called quarks.

Internal structure of all hadrons (strongly [redacted] interacting particles) can be expressed in terms of 3 quarks (u, d, s) with following properties:

	Quark	Charge	Spin	Baryon No	I_3	s	c	t	b
Quarks are I[st] generation	[redacted]	$+2/3\,e$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	$+\left(\frac{1}{2}\right)$	0	0	0	0
	[redacted]	$-1/3\,e$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	$-\left(\frac{1}{2}\right)$	0	0	0	0
II[nd] generation	[redacted]	$-1/3\,e$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	0	s	0	0	0
	Charm (c)	$+\frac{2}{3}e$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)$	0	0	c	0	0

For antiquarks, charge, B are opposite in sign.

III[rd] generation	top (t)	$+\frac{2}{3}e$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	0	0	0	t	0
	bottom (b)	$-\frac{1}{3}e$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	0	0	b	0	b

The values of s, c, t, b are calculated from Modified Gellman Nissigema Formula.

For Resonance Particle, "Upsilon Meson" was discovered.

To describe it, ~~[illegible]~~ t & b were introduced.

$$\boxed{Q = I_3 + \frac{1}{2}(S + B + C + t + b)}$$

For Strange Quarcke

$$-\frac{1}{3} = 0 + \frac{1}{2}\left(-1 + \frac{1}{3} + C + 0 + 0\right)$$

$$\Rightarrow -\frac{1}{3} = -\frac{1}{2} + \frac{1}{6} + C$$

$$\Rightarrow -\frac{1}{3} = \frac{-1}{3} + C \quad \Rightarrow \boxed{C = 0}$$

For up Quark

$$\frac{2}{3} = I_2 + \frac{1}{2}\left(0 + \frac{1}{3} + 0 + 0 + 0\right)$$

$$I_2 = \frac{2}{3} - \frac{1}{6} = \frac{3}{6} = \left(\frac{1}{2}\right)$$

$$\boxed{I_2 = \frac{1}{2}}$$

For Strange Quark

$$-\frac{1}{3} = 0 + \frac{1}{2}\left(S + \frac{1}{3}\right)$$

$$\Rightarrow -\frac{2}{3} = S + \frac{1}{3} \quad \Rightarrow \boxed{S = -1}$$

For charm quark

$$\frac{2}{3} = 0 + \frac{1}{2}\left(0 + \frac{1}{3} + c\right) \Rightarrow \boxed{c = 1}$$

Internal structure of Mesons

For them, Spin = 0 ✓

These are the most important property of Mesons

Also, B = 0 ✓

3 quarks have never make 0 spin.

✓ [Internal structure of Meson is written in terms of 1 quark & 1 antiquark]

(★) Note that almost all the normal particles are expressed in u, d, s or their antiparticles $\bar{u}, \bar{d}, \bar{s}$

π Mesons

π^+ : { Spin 0, B 0, Charge +1, Strangeness 0 } ⇒ $u\bar{d}$ — Only possible

No strange quark.

$I_3 = +1$

unique def^n of π^+ meson

Q B S is definition of particle

I_3 : opposite sign in case of antiparticle

π^- : $\bar{u}d$ — { S = 0, B = 0, Q = −1, S = 0 }

π^0, η^0 ⇒ both { S = 0, B = 0, Q = 0, S = 0 } — $u\bar{u}$ or $d\bar{d}$

$$\psi = \frac{1}{\sqrt{2}}\left[u\bar{u} \pm d\bar{d}\right]$$

Eta Meson

Pi Mesons have odd parity

It happens when anti symmetric wave function.

$\Rightarrow \quad \pi^0 : \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

Hence

$\eta^0 : \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$

K Meson

$K^+ : \left\{ S=0,\ B=0,\ s=+1,\ Q=+1 \right\}$

$K^0 : \left\{ S=0,\ B=0,\ s=+1,\ Q=0 \right\}$

$K^+ : u\bar{s}$

$K^- : \bar{u}s$

$K^0 : d\bar{s}$

$$\eta^0 : \left[\frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}\right]$$

$$\pi^0 : \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

Internal Structure of Baryons

Nucleons

Neutron n
$$\left\{ \begin{array}{l} Q = 0 \\ B = +1 \\ S = 0 \\ \text{Spin} = \frac{1}{2} \end{array} \right\}$$

udd

Antineutron $= \bar{u}\bar{d}\bar{d}$

Problem with Λ^0 and Σ^0
$$\left\{ \begin{array}{l} Q = 0 \\ B = +1 \\ S = -1 \\ \text{Spin} = \frac{1}{2} \end{array} \right\}$$

$\psi_1 = uds$

$\psi_2 =$

→ We will look for intrinsic parity

For Odd $\Rightarrow \frac{1}{\sqrt{2}}(\psi_1 - \psi_2)$

Even $\Rightarrow \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$

Proton p

uud

Antiproton $= \bar{u}\bar{u}\bar{d}$

Omega (Ω^-) :
$$\left\{ \begin{array}{l} Q = -1 \\ B = +1 \\ S = -3 \\ \text{spin} = 3/2 \end{array} \right\}$$

sss

$\uparrow\uparrow\uparrow$

Problem as Pauli Exclusion Principle is violated....

Remember this example!!!

Xi

Ξ^-
$$\left\{ \begin{array}{l} Q = -1 \\ B = 1 \\ S = -2 \\ \text{spin} = \frac{1}{2} \end{array} \right\}$$

dss

$\uparrow\downarrow$

Ξ^+ $\bar{d}\bar{s}\bar{s}$

Ξ^0 uss

To resolve this discrepancy, we have introduced concept of coloured quarks.

Equal combination of red, blue & green. ⇒ all observed particles are colourless.

Every Quark comes in 3 colours: red, blue & green.

⇒ (SSS) are all different colours.

$S_R S_B S_G : \Omega$

All colours have anti colours too.

Strong Interactions arises due to colour. Colour plays same role in strong interaction what charge plays in EM interactions. (same colour → repulsion)

Strong Interaction b/w coloured quarks occur via exchange of gluons. Its called Quantum Chromodynamics.

Red - Red → good interaction
Blue - Blue → good interaction
Green - Green → good interaction

12 gluons have been proposed.

Reaction which obey conservation laws but are not observed. To explain that in 1971, Mainani introduced Charm Quark

To explain (short lived resonance particles called Upsilon Mesons) in 1979, top & bottom quarks were introduced.

$^{23}_{11}Na$	n	udd
	p	uud

$Z = 11$ 22 u and 11 d

$N = 12$ 12 u and 24 d

34 u & 35 d ✓

Idea about Unification

4 fundamental interactions.

We have unified 2 interactions

Combined Electroweak Interaction ..... interaction b/w leptons & charged particles.

Weinberg & Salaam Model ......

Force arises due to exchange of particles.

Snatched : attraction

thrown : repulsion

Electroweak Interaction arises due to exchange of 4 bosons : $W^{\pm}$, Z^0, photon via phenomenon called Spontaneous Symmetry Breaking (SSB)

3 of the bosons acquire huge mass and 4th becomes massless in this phenomenon. Massless part is photon, travels with $v = c$, and its responsible for ∞ range of EM interaction and other particles are responsible for weak interactions. $\Rightarrow$ Range become extremely small as mass is

$$\left(\frac{80{,}000\ MeV}{c^2}\right) \quad W^{\pm}$$

$$\left(\frac{90{,}000\ MeV}{c^2}\right) \quad Z^0$$

$W^{\pm}$ and Z^0 ← intermediate vector bosons

These exchange particles are quanta of Electroweak Interactions.

(★) "Strong force is spin dependent" is also known from the fact that ortho and para Hydrogen (H_2), (2 spin state combination of 2 protons) have different cross section for neutron scattering experiment.

Ortho & Para H_2 :

Ortho   Para

2 proton spins aligned parallel / antiparallel.

at room temperature : 3 : 1

Nuclear ~~Particle~~ Physics (4)

- Deuteron
- Fusion Cycles

Quadrupole Moment

('s' orbital के लिए 0)

$$Q = \left(\frac{2I-1}{2I+2}\right) R^2$$

where $R^2 = \frac{3}{5} R_0^2 A^{2/3}$

→ For a shell model, those nuclei with closed shells should be spherically symmetric and $Q = 0$

Examples of Shell Model Application

- 10 isotopes of $_{50}Sn$ ⇒ abundance at $Z = 50$
  ⇒ stability around magic numbers.

  2, 8, 20, 28, 50, 82, 126, ...

- $_{82}Pb$: 7 nuclei at [redacted] $Z = 82$.
- ^4_2He : very stable

✔ $19 < Z < 37$: island of isomerism.
large no. of nuclear isomers. It amounts to transition : $f_{\frac{7}{2}}$ to $p_{\frac{1}{2}}$

Nature of Strong Nuclear Force

- We have done all characteristics, Yukawa's Meson Theory along with feynmann diagrams, exchange theory.
- We have to do deuteron problem:
  - Ground state of deutron
  - Magnetic Moment

Deuteron

(★) Deutron Problem in 2 steps
(A) Theoretical Modelling
(B) Quantum Solution

Deuterium ^2_1H 1 p and 1 n and 1 e

Remove e^-. We get only nucleus. Nucleus of Deuterium is called Deuteron.

There is no other simple nucleus than deuteron to study nuclear strong force.

There are 7 important characteristics of deuteron.

(1) It is loosely bound [low Binding Energy] structure of 1N and 1P typically observed in ground states.

(2) Precisely known value of B.E. = 2.225 MeV

(3) Size 'b' fm precisely known b = 4.2 fm

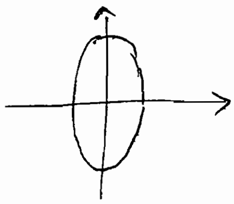
For comparison, B.E. of α particle is 28.3 MeV

(4) Nuclear Spin I = 1

empirically known. Can be seen from nordham strong rule

(5) Magnetic Moment $(\mu)_z = 0.8574\ \mu_N$

(6) Quadrupole Moment $Q = +0.00282 \times 10^{-28}\ m^2$ (Prolate spheroid)



(7) Parity of deuteron is even

Size of deuteron is bigger while range of force is small ⇒ loosely bound structure.

Ground state of deuteron is of triplet state

$S = 1;\ r = 2S+1 = 3$

$^3S_1$

→ If $Q \neq 0$ ⇒ cannot be spherically symmetric

Hence some distortion has occured with spherically symmetric $^3S_1$ state.

$$\vec{I} = \vec{L} + \vec{S}$$

starting from here

✓ We know $I = 1$ ①

$\vec{L} = \sum_i \vec{l_i} = \vec{l_1} + \vec{l_2}$

$\vec{S} = \sum_i \vec{s_i} = \vec{s_1} + \vec{s_2}$

$|s_1 - s_2| < s < |s_1 + s_2| \Rightarrow$ $\boxed{0 < s < 1}$

✓ Not observed in excited state .... most probably $l = 0$ ②

$\vec{L} = \vec{I} - \vec{s}$

$\Rightarrow |i - s| < l < |i + s|$

If $s = 1$ ↑↑

$\Rightarrow 0 < l < 2$ $l = 0, 1, 2$

① and ② ⇓ ③

If $S = 0$, ↑↓ $L = I = 1$ ⇒ excited state always

Hence $S \neq 0$ ③.

From above analysis, we can see that the force keeping the 2 nucleons together is spin dependent.

✓ Hence along with $I = 1$, only S acceptable is $S = 1$ ③

1st conclusion

Quantum Mechanics Principle: If given potential function is symmetric, this implies ψ is either of odd/even parity but not together.

$\psi_1 : l = 0, 2$ (ground) even parity

$\psi_2 : l = 1$ (excited) odd parity

Even and odd values of l cannot exist in the same wavefunction ψ in order to conserve parity

$\Rightarrow \psi$ valid $\Rightarrow l = 0, 2$. <u>2nd conclusion</u>

$$\psi = C_1 \psi(1s) + C_2 \psi(1d)$$

(low Prob in excited state)
due to low value of C_2, we have little departure from sphericity.

* क्या यही logic है ?!!
ψ is combination of odd-odd or even-even eigenfunctions, and For ground state we know that $l=0$ is part of ψ $\Rightarrow \psi = \psi(l=0) + \psi(l=2)$

$(\mu_p)_z = +2.793\ \mu_N$

$(\mu_N)_z = -1.91\ \mu_N$

$\mu_{deuteron}$ in ground state

$(\mu_d)_z = +0.883\ \mu_N$

Observed value has discrepancy.

$(\mu_d)_{z\,observed} = 0.8574\ \mu_N$

if $l=0 \Rightarrow$ Contribution of only spins in moment $\neq$ observed value $\Rightarrow l=2$ will also be there

Hence departure from $Q=0$ & $(\mu_d)_z$ discrepancy gives (proof of) excited state of $C_2 \psi(1d)$. <u>3rd conclusion</u>

$$\psi = C_1 \psi(1s) + C_2 \psi(1d)$$

$$|C_1|^2 + |C_2|^2 = 1$$

$$|C_1|^2 \simeq 0.96$$
$$|C_2|^2 \simeq 0.04$$

<u>4th conclusion</u>

It reconciles μ_z and Q.

✓ Force between neutron and proton is spin dependent. (5th conclusion)

✓ Also, it transits from 1s to 1d state ⇒ angular momentum is not a const. of motion (6th conclusion)

⇒ not a central force. Also $Q \neq 0$ ⇒ Not symmetrical force.

Hence Nuclear Force is not a central force, i.e. it is a Tensor Force.

Neutron & Proton create a strong potential field, V_0.

Quantum Mechanical / Wave Mechanical Model of Ground State of Deuteron

Ground state : $l = 0$ ⇒ spherically symmetric ⇒ $\psi = f(r)$ only

$$H\psi = E\psi$$

$$H = \frac{p^2}{2m} + V$$

$$= -\frac{\hbar^2}{2m}\nabla^2 + V$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

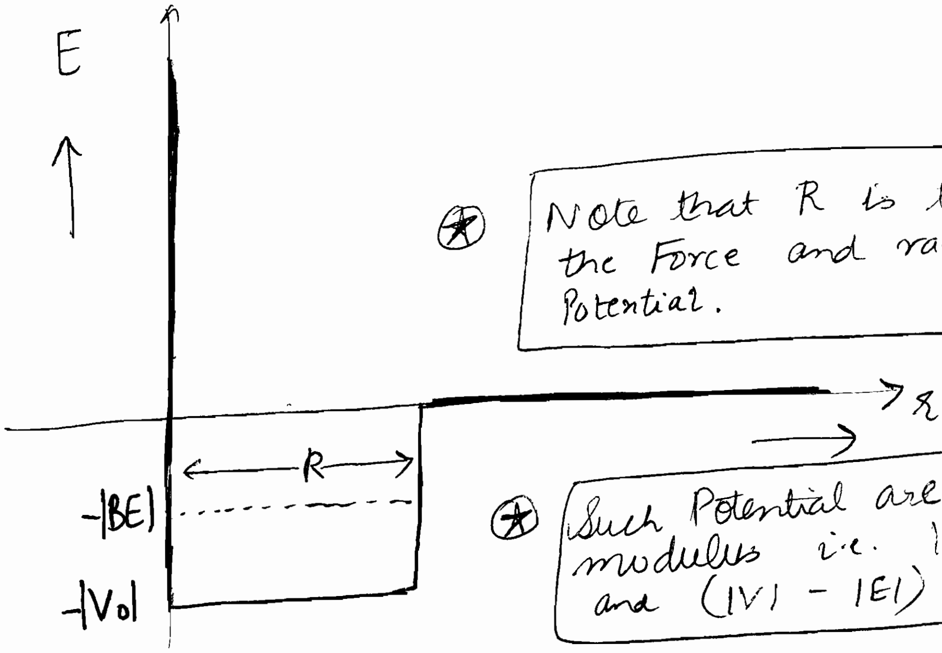
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{(Spherically Symmetric)}$$

Put $\psi(r) = \frac{u(r)}{r}$

$$\Rightarrow \boxed{\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}(E-V)\,u = 0}$$

$$\psi(r) = \frac{u(r)}{r}$$

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}(E-V)u = 0$$



⊛ Note that R is the range of the Force and range of the Potential.

⊛ Such Potential are tackled via modulus i.e. |V| and |E| and (|V| - |E|) etc.

NO one knows what is Potential well of Strong Force is like. We can assume it to be Harmonic Potential or ~~simplest~~ well. Yukawa Potential. We assume to be ~~simplest~~ well.

From deuteron model, we know $F_{attractive} \simeq 35$ MeV.

- R is range of force. It is not equal to radius of dueteron. Force may not operate within whole radius.

$B.E. < 0$

$$V_0 < E < 0$$

At 1st approximation, we can take $R = b$ (radius of nucleus)

For $r < R$

$$\frac{d^2u_1}{dr^2} + \underbrace{\frac{2m}{\hbar^2}(-V_0+B)}_{k_1^2} u_1 = 0$$

Note that Potential w.r.t. Cartesian Coordinates
⇒ Even/Odd Parity ~~but~~ $\psi(r)$ not symmetric

$$\frac{d^2u_1}{dr^2} + k_1^2 u_1 = 0$$

$$u_1(r) = A\sin(k_1 r) + B'\cos(k_1 r)$$

At $r = 0$, ~~ψ is finite~~ $u(r) = r\psi(r) = 0$

$$\Rightarrow \boxed{u_1(r) = A\sin(k_1 r)}$$

and $B' = 0$

$$k_1 = \sqrt{\frac{2m}{\hbar^2}(|V| - |E|)}$$

For $r > R$

$$\frac{d^2u_2}{dr^2} + \underbrace{\frac{2m}{\hbar^2}(-B)}_{k_2^2} u_1 = 0$$

$$\frac{d^2u_2}{dr^2} - k_2^2 u_2 = 0$$

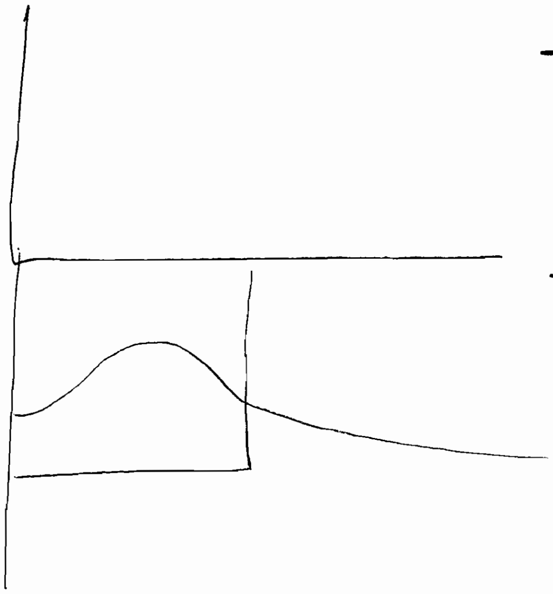
$$\Rightarrow u_2(r) = Ce^{k_2 r} + De^{-k_2 r}$$

At $r = \infty$, ψ is finite

$$\Rightarrow \boxed{u_2(r) = De^{-k_2 r}}$$

and $C = 0$

$$k_2 = \sqrt{\frac{2m}{\hbar^2}|E|}$$



→ अरे भाई, जहाँ तक Potential है, वहाँ तक range of force है !! (atleast existence of 1 solution से R_{min} निकाला)

→ जहाँ तक Probability of occurrence sufficient है (penetration depth), वहाँ तक radius है !!
(atleast radius $= \frac{1}{k_2}$)

at $r = R$, $u_1(r) = u_2(r)$, $\left.\frac{du_1}{dr}\right|_{r=R} = \left.\frac{du_2}{dr}\right|_{r=R}$

$\Rightarrow A \sin k_1 R = D e^{-k_2 R}$

$A k_1 \cos k_1 R = D(-k_2) e^{-k_2 R}$

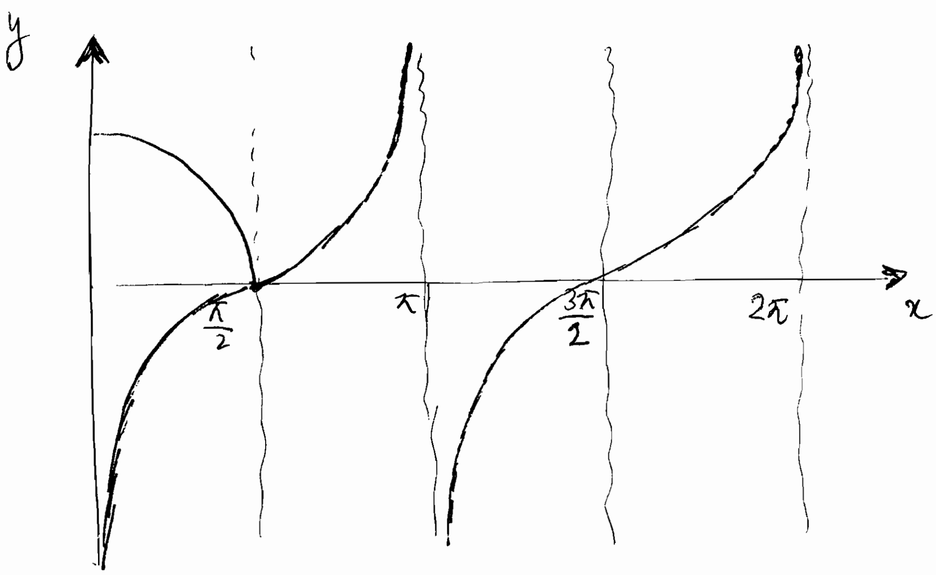
$\Rightarrow k_1 R \cot(k_1 R) = -k_2 R$

$k_1 R = x$

$k_2 R = y$

$\Rightarrow \boxed{x \cot x = -y}$ $\Rightarrow \boxed{y = x \tan\left(\frac{\pi}{2} + x\right)}$

Also $\boxed{x^2 + y^2 = \frac{2m}{\hbar^2} |V_0| R^2}$



I am interested in +y values only.

At least for 1 bound state, $\sqrt{\eta^2+\xi^2} \geq \left(\frac{\pi}{2}\right)$

$$\frac{2m}{\hbar^2}|V_0| R^2 \geq \frac{\pi^2}{4}$$

$$\Rightarrow |V_0| R^2 \geq \left(\frac{\hbar^2\pi^2}{8m}\right)$$

$$\boxed{(V_0 R^2)_{min} = \left(\frac{\hbar^2\pi^2}{8m}\right)}$$ ✓

Assumptions :-

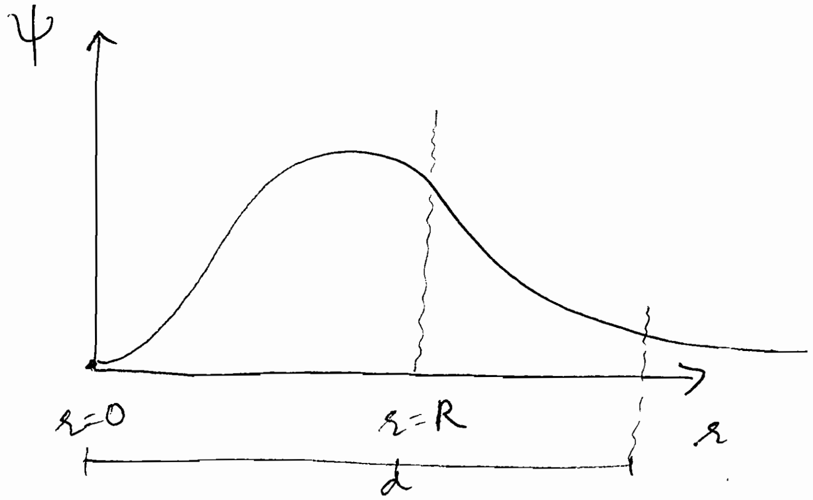
① 1st assumption, $R = b$

i.e. range of force is acting

(We assumed Probability of finding the particle drops after $r = R$)

② $m = \left(\frac{m_p m_n}{m_p + m_n}\right) \simeq \left(\frac{m_n}{2}\right)$ ✓

2 body to 1 body Problem



We can assume Penetration depth till value of ψ drops to $\left(\frac{1}{e}\right)$ of its maximum value in IInd region

$$De^{-k_2 d} = De^{-1} \Rightarrow \boxed{d = \frac{1}{k_2}}$$

Radius of deuteron $= \left(\frac{1}{k_2}\right) =$ Penetration depth of ψ

$$r_d = \left(\frac{1}{k_2}\right) = \frac{\hbar}{\sqrt{2m|B|}}$$

radius of nucleus — till Penetration depth

i.e. till significant Prob. of finding partic

$$R_{min} = \frac{\pi \hbar}{\sqrt{8m|V_0|}}$$

minimum range of force

$$\frac{r_d}{R} = \frac{\sqrt{8m|V_0|}}{\pi\sqrt{2m|B|}} = \frac{2}{\pi}\sqrt{\frac{|V_0|}{|B|}}$$

$V_0 \simeq 30$ MeV (estimate)

$$\frac{r_d}{R} = \frac{2}{\pi}\sqrt{\frac{V_0}{B}}$$

$$\frac{r_d}{R} = \frac{2}{\pi}\sqrt{\frac{30}{2.225}} = 2.338$$

$$\Rightarrow \frac{r_d}{R} = 2.34$$

$$\Rightarrow R = \left(\frac{r_d}{2}\right)$$

we know r_d from experiments

Hence, range of strong force is half of radius

$\Rightarrow$ loosely bound structure.

★ another explanation of loosely bound structure :-
B.E of deutron is small. Its value is **2.225 MeV** while the energy reqd. to eject a nucleon from nucleus of medium mass number is about 8 MeV.

RADIUS: $r_d \simeq 4.2$ fm $\Rightarrow$ RANGE OF FORCE: $R \simeq 2.1$ fm

✓ Between 2 proton, Collumb energy $\simeq 0.1 MeV$

Explanation of fusion on basis of Binding Energy, we know

$kT \simeq 0.1 MeV$

$T \simeq 10^9 k$

NUCLEAR FUSION

→ This temperature is not available on Earth.

Actually its a Potential Barrier. Quantum Tunneling may hold key to fusion w/o reaching such a high Temp.

But there are ambient temperatures in the cores of stars. 'Thermo Nuclear Fusion' is therefore other name for nuclear fusion reactions.

* KE to overcome Potential of $\frac{e^2}{4\pi\varepsilon_0 R}$ $\approx 0.5 MeV$ $= \frac{3}{2} kT$ $\Rightarrow T \approx 10^9 k$ but Tunneling allows fusion @ $T = 10^7 k$ i.e. SUN temp.

These Nuclear Fusion reaction Release large amount of energy (remember the B.E. curve)

2 cycles of nuclear fusion in course of stars.

Proton – Proton cycle

(early phase of stars)

[redacted]

Mass < 1.5 (Mass)sun

$2 \times \quad {}^1_1H + {}^1_1H \longrightarrow {}^2_1H + {}^0_1e^+ + \nu_e + Q_1$

$2 \times \quad {}^2_1H + {}^1_1H \longrightarrow {}^3_2He + Q_2$

$1 \times \quad {}^3_2He + {}^3_2He \longrightarrow {}^4_2He$ [redacted] $+ 2\,{}^1_1H$ [redacted] $+ Q_3$

$4({}^1_1H) \longrightarrow ({}^4_2He) + 2({}^0_1e^+) + 2(\nu_e) +$

24.7 MeV

Carbon nitrogen oxygen cycle (mature star)

CNO cycle

Mass > 1.5 Mass(sun)

Occurs in later stages in life of a star, ~~before dying~~ or in stars with huge mass

[White Dwarf : huge mass has been compacted into small volume

⇒ density ↑↑

& T ↑↑

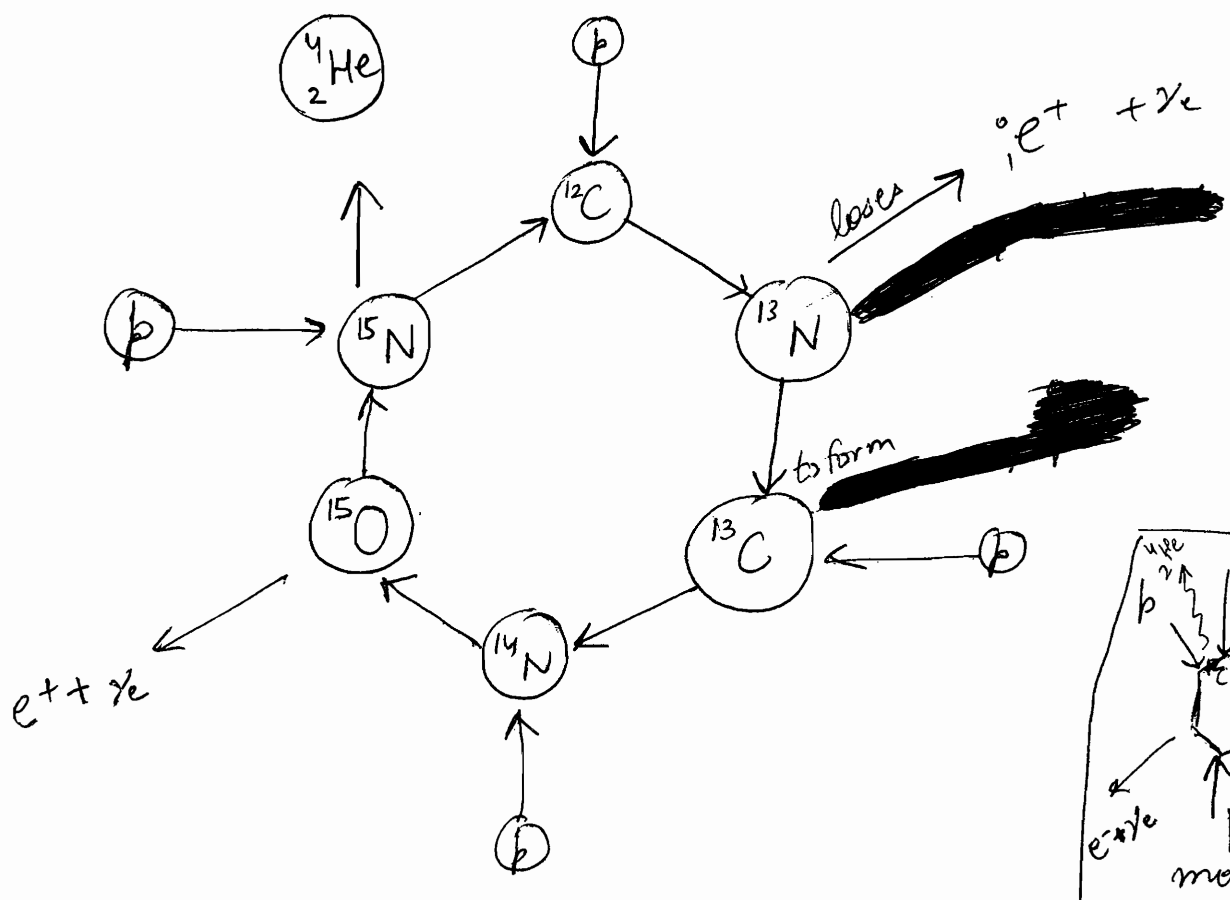
Occurs in white dwarfs, hotter than sun]

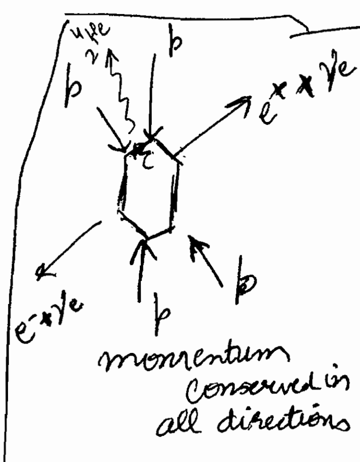
Source of their energy is Carbon - Nitrogen Cycle.

They (C and N) act as catalysts.

Final equation is same

$$4\left({}^{1}_{1}H\right) \xrightarrow{C/N/O} {}^{4}_{2}He + 2e^{+} + 2\nu_e + 24.7\ \text{MeV}$$





Nuclear Physics (5)

06/03/12
- Parity Violation
- β-decay
- Neutrinos

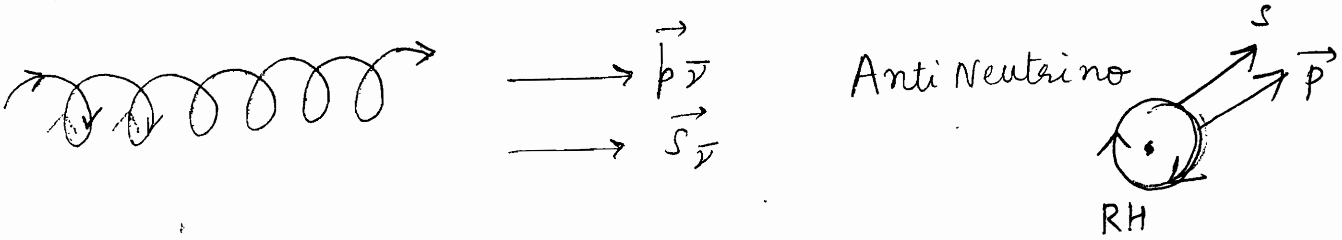
Parity Violation in β-decay

✓ Behaviour of ψ under inversion of coordinates is described by Parity.

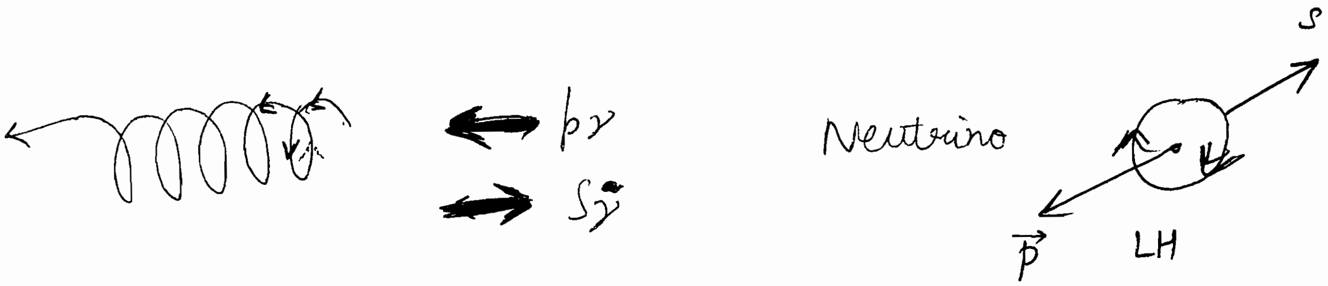
Concept of Helicity

✓ In odd parity, Left handed coordinate system changes to Right Handed Coordinate System under inversion of coordinates

✓ Neutrino & anti-neutrino have difference of LH and RH coordinate system. This is called Helicity.



Helicity = +1



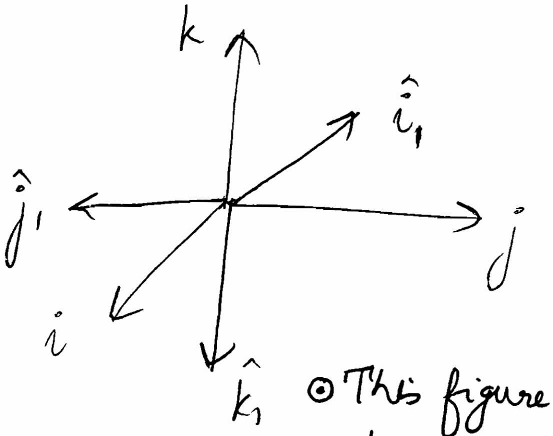
Helicity = −1

- → LH and RH dependent upon circular motion and $\vec{P}$

Helicity is the component of $\vec{s}$ along $\vec{p}$

$$\boxed{H = \hat{S}\cdot\hat{p}} = \frac{\vec{S}\cdot\vec{P}}{|\vec{S}||\vec{P}|}$$

- → $\vec{S}$ is dependent only upon circular motion via right hand screw rule.
- → No role of charge



- The 2 coordinate system differ in handedness :-

$(x, y, z) \longrightarrow (-x, -y, -z)$

RH Parity → LH Parity

- This figure shows that inverting the sign of unit vectors is equivalent to inverting the handedness

$\vec{i}_1 = -\vec{i}$
$\vec{j}_1 = -\vec{j}$
$\vec{k}_1 = -\vec{k}$

✓ If particle appears to be same, then even parity

✓ If particle appears opposite, then odd parity.

It is similar to seeing particle in a mirror $(-x, -y, -z)$.

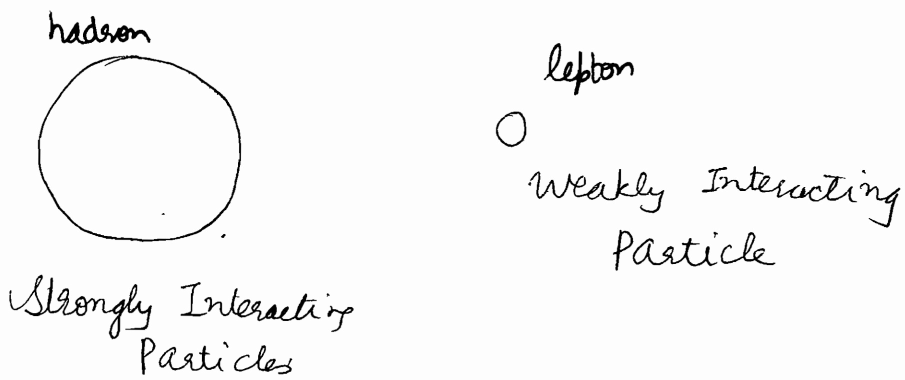
[If same equation of motion in mirror ⇒ even parity.
otherwise we say parity is violated]

Initially, we thought nature is symmetric ⇒ parity should be conserved in all nuclear reactions

But when we observed β- decay. It had different behaviour from then known Strong, EM or Gravity interactions. It was called Weak Interaction. Neutrino is a particle that takes part in all weak interactions.

$> 10^{-8}$ second : maximum time duration of interaction

⇒ low probability of this interaction.



Hadrons are assigned intrinsic parity

$p_p = +1$
$p_n = +1$

→ Proposed by Lee & Yang that parity is not conserved in weak interactions.

Wu and coworkers performed experiment on $^{60}_{27}Co$ nucleus which proved Parity violation in β-decay.

$$^{60}_{27}Co \longrightarrow ^{60}_{28}Ni + e^- + \bar{\nu}_e$$

(even parity) (even parity)

→ Q conserved → L_e conserved

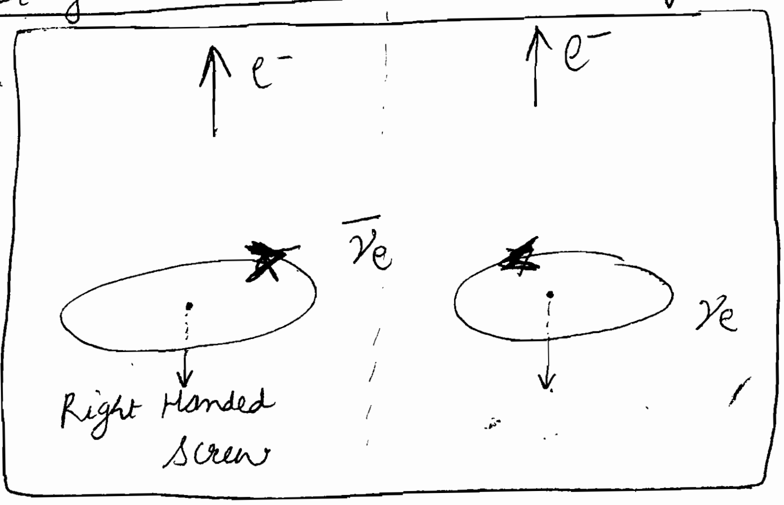
→ A conserved

This reaction occurs only via Weak Interaction force.

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

remember neutron and proton have even parity....

To have Parity Conservation, e^- and $\bar{\nu}_e$ must have same parity. By appling $\vec{B}$, I can have motion of e^- in particular direction. Now I see ~~it~~ in mirror. Behaviour of ν in mirror gets reversed due to helicity. Hence parity is not conserved.



Perfect diagram for explanation

Modern Physics : It starts from theory of relativity and Planck Quantum Theory. 4 experiments are landma[rks] in Modern Physics.

1) Frank-Hertz Experiment (1912-13)

Energy level of H atom are discrete.

2) Davisson-Germer Experiment

De Broglie concept of material waves.

3) Stern-Gerlach Experiment

Spin Concept

4) Lee-Yang & Hu Experiment

β-decay.

(2), (3), (4): in syllabus — every year 2 are asked out of 3.

β Decay

β decay or α decay or γ decay are ways of nuclei to achieve stability. These 3 are performed inside the nucleus.

$$^{A}_{Z}X \rightarrow \,^{A'}_{Z'}Y + \beta \leftarrow \text{electron/positron}$$

or

$$\gamma \leftarrow \text{photon}$$

In β decay, A is conserved

There are 3 types of β decay reactions :

① $^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$ } β particle emission

i.e. $n \rightarrow p + e^-$

② $^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+$ } Positron emission

i.e. $p \rightarrow n + e^+$

③ $^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y$ } k-shell e^- capture

$p + e^- \rightarrow n$

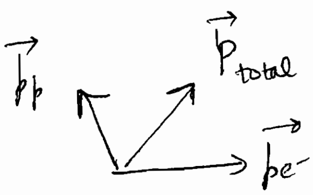
Physics of Neutrinos

Pauli observed that such spontaneous reactions are not possible due to :

(i) $\vec{p}_p + \vec{p}_{e^-} = 0$ for 1st reaction as $\vec{p}_n = 0$ (initially)

$\Rightarrow \vec{p}_p = -\vec{p}_{e^-}$

It was observed that they are not at 180°



i.e. $\vec{p}_{total} \neq 0$

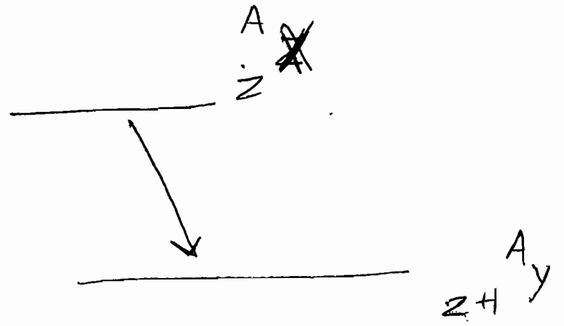
Hence conservation of linear momentum is violated

(ii) Spin Conservation is violated (Conservation of Angular Momentum)

$$\frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$$

(iii) $${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$$

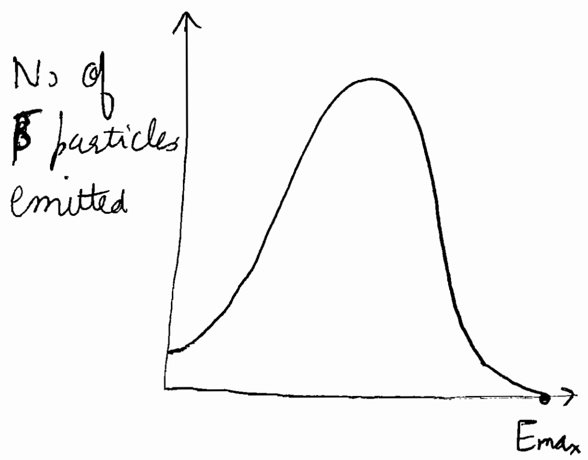
(Parent) (Daughter)



Applying total relativistic energy conservation,

The difference of energy of parent & daughter must be energy of electron ⇒ All e⁻ have same energy. But we do not observe singe value of Energy. We observe Spectra.

→ E_{max} is characteristics of nucleus, called end point energy. $(E_\beta)_{max}$



Continuous Spectrum of β decay

We can observe Some features from it :

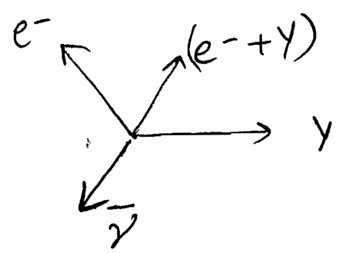
① Energy is continuously distributed b/w E=0 and end point energy.

② β-particles are maximum with mid-Energy level.

⇒ relativistic energy conservation is violated.

Hence; its not a 2-body problem. It must be a 3 body problem.

[Hence we require neutral particle of $\left(\frac{1}{2}\right)$ spin along with other 2 products]



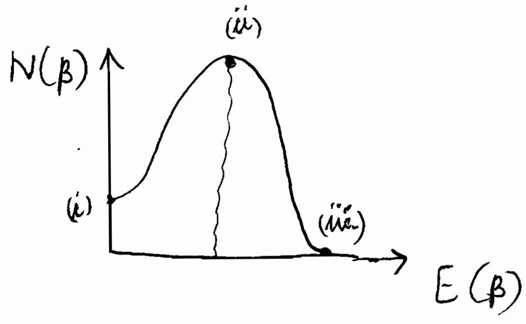
$\vec{p}_{total} = 0$

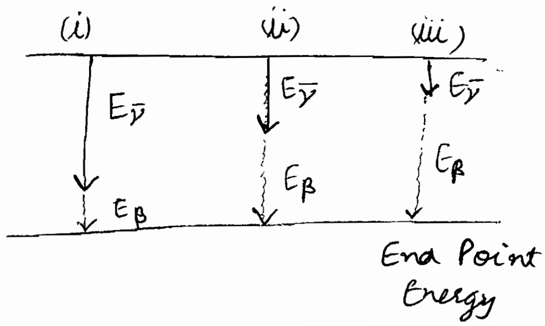
$\frac{1}{2} \longrightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ ⇒ S conserved

(half integral) (half integral)

Relativistic Energy difference 'ΔE' is shared between e⁻ and $\bar{\nu}$

$$E_{\nu}^2 = (p_\nu c)^2 + \cancel{(m_\nu c^2)^2} \simeq (p_\nu c)^2$$





~~Pauli named the 3rd particle as Neutrino.~~

✓ From lepton number conservation we get the particle as Anti Neutrino

✓ Pauli proposed that there should be a 3^{rd} neutral particle of $\left(\frac{1}{2}\right)$ spin.

✓ Fermi used it to make quantum mechanical theory of these particles, whom, Fermi called Neutrinos.

He developed a golden rule regarding "Time dependent Perturbation Theory," called Fermi Theory of β decay.

[changes in Hamiltonian]

$$^{A}_{Z}X \rightarrow \, ^{A}_{Z+1}Y + \beta^- + \bar{\nu}_e$$

$$^{A}_{Z}X \rightarrow \, ^{A}_{Z-1}Y + \beta^+ + \nu_e$$

$$p + e^- \longrightarrow n + \nu_e$$

According to fermi, β-decay is explained by a 3^{rd} Particle which exists in 2 states : Neutrino & Anti Neutrino.

He applied Quantum Mechanics to obtain Probability of transition.

$$\left\{ \begin{array}{l} i \rightarrow f + e^- + \bar{\nu}_e \\ i \rightarrow f + e^+ + \nu \\ i + e^- \longrightarrow f + \nu_e \end{array} \right\}$$

$$P(i \to f) = \frac{2\pi}{\hbar} \left| \psi_{i \to f} \right|^2 \rho_{if}$$

Fermi's Golden Rule

where ρ_{if} = density of state $= \left(\frac{dN}{dE}\right)$

& $\psi_i : \psi_{Parent}$

& $\psi_f : \psi_{daughter} \, \psi_{e^-} \, \psi_{\bar{\nu}_e}$

- ✓ ft parameter is calculated.
  Since ft is huge, we talk in terms of $\log(ft)$

- ✓ We also have Fermi Selection Rules
  - ✓ Nuclear Parity should not change. (overall parity violation may be there but $P_{parent} = P_{daughter}$) $P_{parent} = P_{daughter}$
  - ✓ Orbital Angular Momenta should not change
  - ✓ $\Delta i = 0$ (for whole reaction) i: nuclear spin

- ✓ Allowed Reactions are having low value of ft parameter
  High $ft \Rightarrow t\uparrow$ (lifetime) $\Rightarrow$ low probability

- ✓ $\Delta l = 1$: 1st order forbidden transition
- ✓ $\Delta l = k$: k^{th} order forbidden transition

~~Gammo Tiller~~ changed $\Delta i = 0, \pm 1$ thus modifying fermi transition rules.

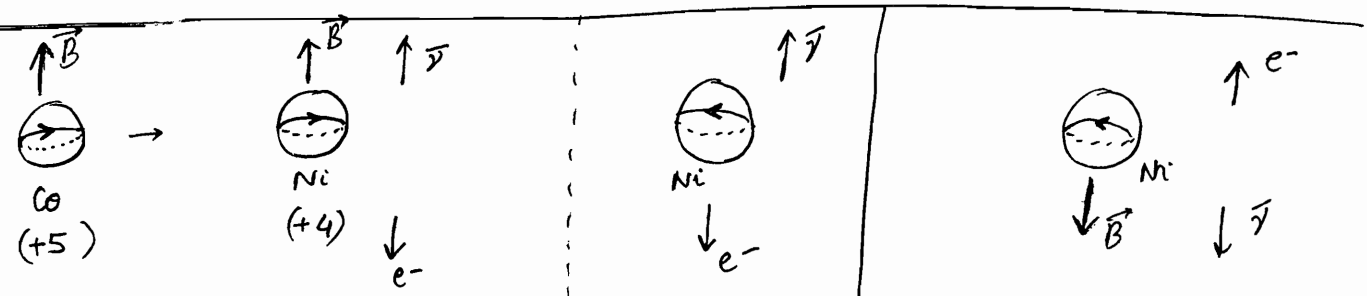
Some key Points

(1) Self sustained fusion reaction occurs in H-bomb. The explosive substance is a mixture of ^2_1D and ^3_1T. The high temperature needed for reaction is produced by exploding a fission bomb. $^3_1H + ^2_1H \rightarrow ^4_2He + ^1_0n + \Delta Q$

(2) Controlled fusion reaction have been achieved using deuterium plasma. At temp of around $10^8 K$, fusion rxn begins to release energy. At this temp., energy escapes as heat through container walls. Hence, strong magnetic fields are used for thermal insulation of the plasma in a Toroidal Magnetic Chamber TOKAMAC.

(3) For a fusion, energy released = 24.7 MeV
If energy due to annihilation of two positrons is also added, then total energy released $\approx 24.7 + 2 \approx 26.7$ MeV per cycle.

(4) 3 properties of Neutrino :
(i) electrically neutral
(ii) mass almost zero ⟶ It follows from the fact that $(E_\beta)_{max}$ observed is equal to mass-energy difference of parent and [product + electron]. If neutrino had finite mass comparable to m_{e^-}, then it must be subtracted to get $(E_\beta)_{max}$
(iii) Spin = (½) ⇒ F.D. statistics



1st fix $\bar{\nu}$ direction as $\vec{p} \parallel \vec{s}$, which $(\vec{s})$ should be aligned with spin of Ni to conserve angular momentum
Now to conserve $\vec{p}$, e- in opposite direction

Mirror Image

[These two are not equivalent]

Nuclear Physics (6)

(6A)

: • Gamma decay 06/03/12

• $ft = \dfrac{2\pi^3\hbar^7 \ln 2}{g^2 c^4 m^5 |M_{if}|^2} = \dfrac{0.693\, \tau_0}{|M_{if}|^2}$ ← time period or lifetime

parameter for β decay

$g = 1.41 \times 10^{-62}$ joule m^3

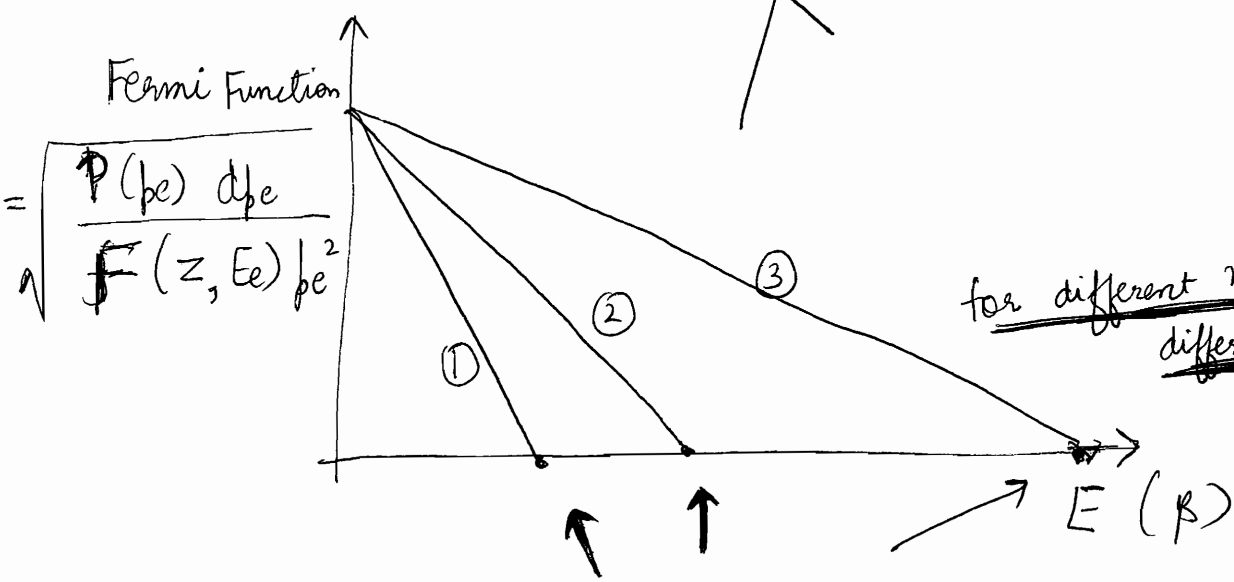
If $\log_{10}(ft) = 2.7$ to 3.7 ⇒ superallowed transitions

if $\log_{10}(ft) = 4$ to 5.8 ⇒ allowed transition

if $\log_{10}(ft) = 6$ to 10 ⇒ 1^{st} forbidden

⋮ → k^{th} forbidden

Kurie's Plot is obtained.



for different nuclei, different curves

end point energies

(E const. ⇒ p const. [If 1γ photon] But p of γ rays will depend on the initial direction of e^- and e^+ ⇒ p is variable ⇒ [2γ photons minimum]

From end point energies, limits on mass of neutrino is obtained. (Beiser)

Another example of β decay:

$${}^{6}_{2}He \rightarrow {}^{6}_{3}Li + e^- + \bar{\nu}_e \quad \text{(check)}$$

$$0^+ \rightarrow 1^+$$

$$0^{(+)} \qquad (3^+)$$

(allowed according to gammo taylor selection rule)

We know

$$n \rightarrow p + e^- + \bar{\nu}_e$$

β decay : β particle emission

Writing opposite $p + e^- + \bar{\nu}_e \longrightarrow n$

$\Downarrow$ we can write

$$p + \bar{\nu}_e \longrightarrow n + e^+$$

inverse β decay (given in Beiser)

Proton can never decay into neutron & positron w/o $\bar{\nu}$ as neutron is heavier.

For showing this reaction, Reines and Cowan got Nobel Prize. This reaction is utilized for indirect detection of anti neutrino. If this reaction occurs $\Rightarrow$ $\bar{\nu}_e$ is present.

$\bar{\nu}_e$ can be obtained from flux of $\bar{\nu}_e$ emitted in β decay reaction. Here, in the vicinity, I keep some protons. e^+ is produced. It annihilates with e^- to emit specific γ EM radiation. We can detect that γ photons.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

★ γ: 0.511 MeV each

→ In the Reines & Covan Experiment, (★) refer P- 495 Arthur Beiser

Aqueous solution of Cadmium Chloride

Cadmium has large absorption cross-section.

Neutron are captured by Cadmium Nucleus and energy absorbed is recorded by spectroscope.

Also annihilation of e^+ and e^- gives out energy which is recorded.

(6B)

<u>Gamma Decay</u>

<u>Internal Conversion</u>

<u>Pair Production</u>

} 3 mutually competing processes.

<u>Gamma rays are highest energy EM radiation.</u>

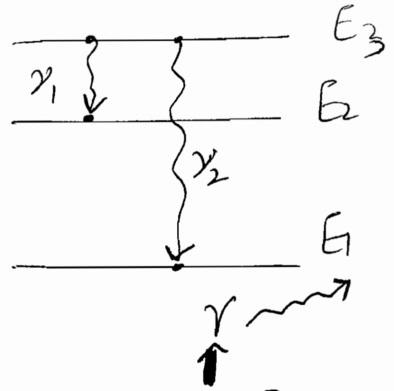
✓ <u>Highest energy X-ray overlaps with lowest energy γ rays.</u>

$$\boxed{E_\gamma > 0.1 \text{ MeV}}$$

✓ <u>Accelaration of charged particles produces EM rays, we know. X-rays are produced by electronic level transitions in tungsten targets</u>

(★) note that these levels are (2J+1) Zeeman type different from states. That were of RF region and used in NMR. (★)

✓ [<u>Gamma rays are arising due to nuclear energy level transitions.</u> Due to some process, say α or β decay, nucleus is left in excited state. When it comes out to ground state, it emits γ photons.]



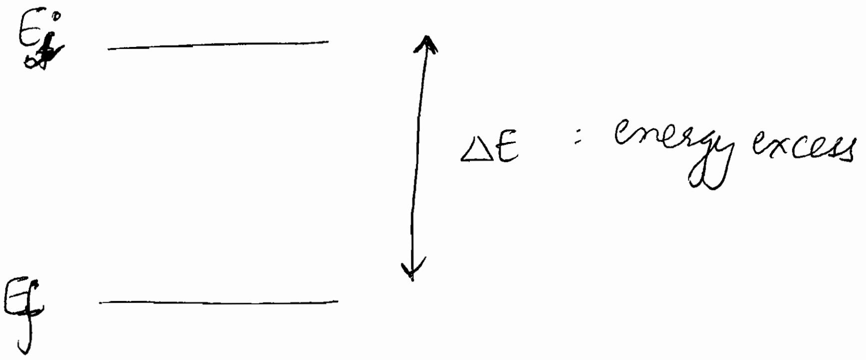
$\gamma_1 = \frac{E_3 - E_2}{h}$

$\gamma_2 = \frac{E_3 - E_1}{h}$

[nuclear photon is absorbed by atomic electron]

Either Gamma Decay or Internal Conversion or Pair Production occurs. They are 3 mutually competing processes. Occuring when nucleus is in excited state.

$\gamma \rightarrow e^- + e^+$



✓ From E_i to E_f, nucleus can come down via,

(i) emitting γ photon (gamma decay)

(ii) ejecting a conversion e^- (internal conversion)

(iii) production of x and $\bar{x}$ (pair production)

eg. $(e^-) + (e^+)$ if $\Delta E > 1.02$ MeV

Internal Conversion

(sort of internally occuring Photo electric effect)

If $\Delta E >$ Binding Energy of e^- from some shell

→ k shell has maximum B.E.

↑ (nearby shell)

[The internal conversion is not a two-step process in which γ-photon 1st emitted & then it knocks out an orbital e^-. Its a single step process in which excited nucleus interacts directly with orbital e^-, transferring its excitation energy to it]

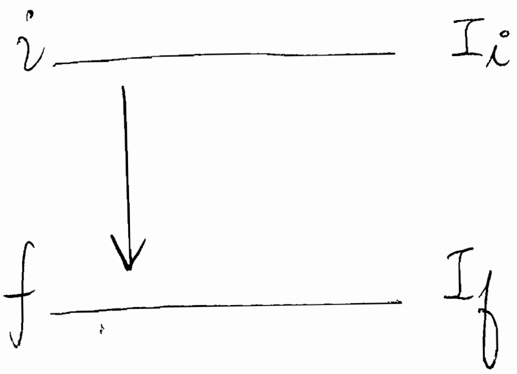
→ K.E. of k-shell e^- = (ΔE − $B.E._{k\text{-shell } e^-}$)

If ΔE is less, we will check for L shell and so on.

Also if k-shell e^- is removed, vacancy is filled by L shell e^-, which is filled by M shell e^- and so on.... Hence we get line spectra. Similar to e^- capture.....

Pair Production

$\Delta E >$ energy required for pair production.



PARITY CHANGE IN GAMMA TRANSITIONS

(★) $I_i \sim I_f$ = Angular Momentum of Photon carried

$\vec{L}_\gamma = \vec{I}_i \sim \vec{I}_f$

$$|(I_i - I_f)| \leq L_\gamma \leq |(I_i + I_f)|$$

in β^- decay, Gamma Tiller conditions for ΔI (ground state of different nuclei). But in γ decay, no such conditions (same but excited nuclei)

Every nucleus has nucleons. Its like a liquid drop.

If 1 p (proton) displaced, dipole created.

If 2 p displaced, quadrupole created

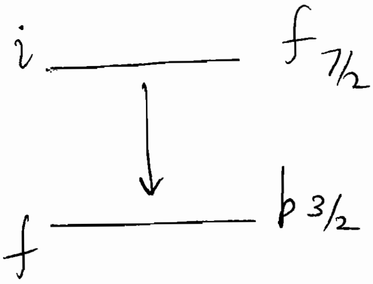
If n p displaced, multipole created.

*this is how excitation takes place

Within nucleus, there is charge flow. these charges are in format of multipole i.e. 2^l

if $l = 0$: monopole
if $l = 1$: dipole
if $l = l$: 2^l multipole.

They can be electric or magnetic multipole.



$I_i = 7/2$

$I_f = \frac{3}{2}$

$L = I_i - I_f$ to $I_i + I_f$
$= 2, 3, 4, 5$

2 ↑ quadropole transition; 5 ↑ multipole transition

Parity change $= P_i P_f = (-1)^L$ for electrical multipole
$= (-1)^{L+1}$ for magnetic multipole

for $l = 2$: quadrupole transition

If electric : $(-1)^2 : 1 :\Rightarrow$ parity should not change

If magnetic : $(-1)^3 : -1 \Rightarrow$ parity changes.

✓ Now I measure parity and tell whether electric or magnetic multipole transition.

For photon to be ejected, L should be minimum 1 as its spin is 1.

DIFFERENT CONDITIONS FOR DIFFERENT GAMMA RAY CONVERSION

If $L < 1 \Rightarrow$ internal conversion as e^- can have spin <1.

But If $\Delta E >$ Energy required for pair production $\Rightarrow$ Pair Prodn.

$$N = N_\gamma + N_i + N_{p.p.}$$

- N_γ ↑ gamma decay emission
- N_i ↑ internal Conversion. e^- emit from i^{th} shell
- $N_{p.p.}$ ↑ Pair Production

$$1 = \left(\frac{N_\gamma}{N}\right) + \left(\frac{N_i}{N}\right)$$

if $\Delta E <$ E required for Pair Production

$\frac{N_i}{N}$: internal Conversion coefficient $= \alpha_i = \left(\frac{N_K + N_L + N_M + N_N \ldots}{N}\right)$

$$= \frac{N_K}{N} + \frac{N_L}{N} + \frac{N_M}{N} + \ldots$$

$$= \alpha_K + \alpha_L + \alpha_M + \ldots$$

↑ ↑ ↑ Partial Conversion Coefficients

$$(\alpha)_K \propto Z^3 \left(\frac{1}{\Delta E}\right)^{L+\left(\frac{3}{2}\right)}$$ (Various studies done upon a

How do we measure energy of γ particles?

Source of γ particles:

p-γ reactions X (p, γ) Y X + p → γ + Y

n-γ reactions X (n, γ) Y X + n → γ + Y

(*) 0.1 MeV to more than 100 MeV : γ Particles

3 mechanism called secondary electron methods to measure E_γ :

(i) Photo electric emission

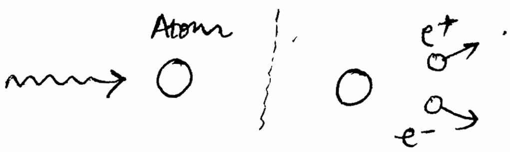
for lowest energy γ photons i.e. $\simeq$ 0.1 MeV
i.e. 100 keV

(ii) Compton Effect

0.1 MeV < E_γ < 1 MeV

(less than pair production)

(iii) Pair Production

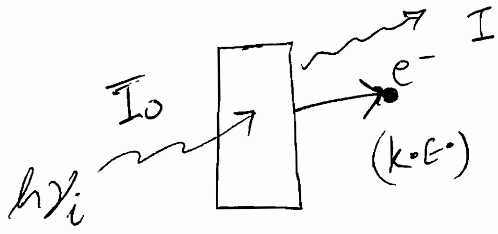


E_γ > 1 MeV

→ Interaction of γ rays with matter is also via these 3 ways.

Photo electric

When suitable energy radiation (E_γ > Work Function) is incident, photo electron is ejected.

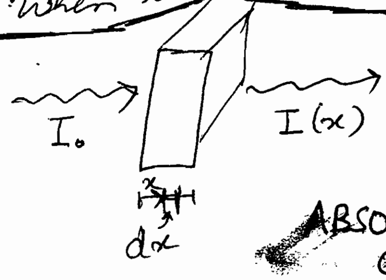


We measure k.E. of electron

$\Rightarrow$ $E_\gamma = h\nu_i$ = Work Function (ϕ) + k.E. $= (\phi + k)$

Hence measured.

Intensity of photons will decrease when they face an absorbing obstruction. $I < I_0$



ABSORPTION OF GAMMA RAYS

$$[dI] \propto [-I\,dx]$$

$$dI = -\mu I\,dx$$

$$\frac{dI}{I} = -\mu\,dx$$

$$\Rightarrow I = Ae^{-\mu x}$$

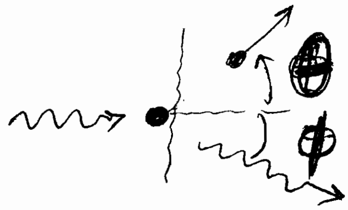
$$\Rightarrow \boxed{I = I_0 e^{-\mu x}}$$

μ: absorption coefficient

$$\boxed{\mu = \frac{1}{x}\ln\left(\frac{I_0}{I}\right)}$$

Compton

$h\nu_i + \quad = h\nu_f + (K \cdot E)_{e^-}$



↑ measured ↓ from

↑ measured $= E_e - m_o c^2$

$\lambda' - \lambda = \frac{h}{m_o C}(1 - \cos\phi)$

$m_o C$: Compton wavelength λ_0

⊙ $\Delta\lambda = \lambda_o [1 - \cos\phi]$

⊙ ϕ : deflection of γ gamma ray

Measure final wavelength (λ') and ϕ we will get (λ)

Pair Production

If $\Delta E > 3$ MeV

$\Rightarrow$ conversion into [e−] and [e+]

or

other pair

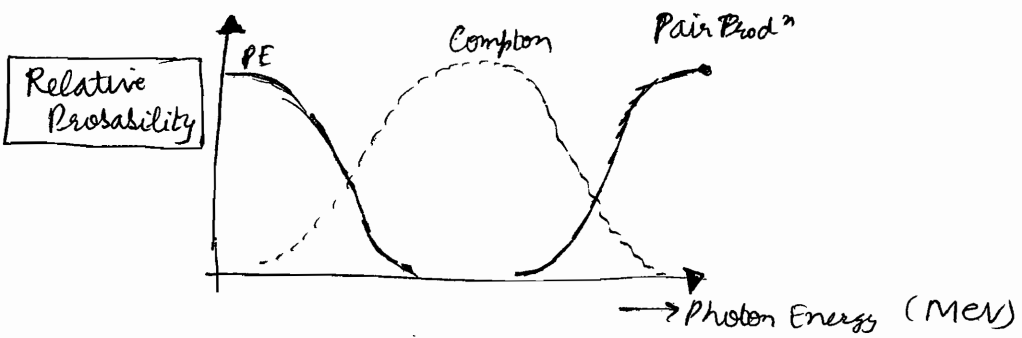
$h\nu_i = E_x + E_{\bar{x}} = 2E_x$

↑ measured

Measure energies of x and $\bar{x}$

(★) In Gamma Decay, A, Z, Energy, $\vec{p}$, Spin will remain conserved.

(★) Remember that spin of photon $= \hbar$



[Remember lecture on Raman Spectra
UV : P.E effect
γ : Compton]

Nuclear Physics (7) : · Mossbauer · Fission 07/03/2012

(7A)

Q What kind of γ - transitions occur ?

(1) $g_{\left(\frac{9}{2}\right)^+} \longrightarrow p_{\left(\frac{1}{2}\right)^-}$

$L = 4, 5$
Parity changing
$\Rightarrow M_4$ & E_5

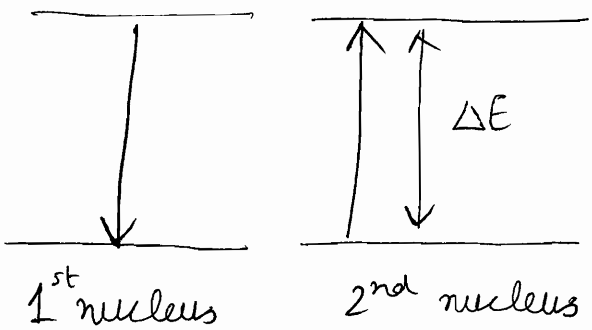
(2) $f_{\left(\frac{5}{2}\right)^-} \longrightarrow p_{\left(\frac{3}{2}\right)^-}$

$L = 1, 2, 3, 4$
↑ ↑ ↑ ↑
M_1 E_2 M_3 E_4
Parity not changing

Moss baeur Effect

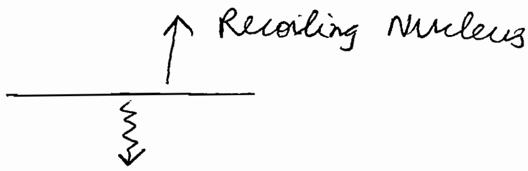
→ It is recoil-less emission & absorption of γ photons.
i.e. Resonant absorption. 'No Recoilness' achieved by trapping nucleus in heavy crystal

→ It provides a source of mono chromatic γ photon
i.e. it resolves very fine lines.



→ It happen when nucleus levels are low lying [redacted], eg [redacted]
10^4 eV (Start of gamma rays)

When γ is thrown out, usually the nucleus recoils.



$$\Delta E - E_{recoil} = E_{\gamma\text{-photon}}$$

$$\nu = \left(\frac{\Delta E - E_{recoil}}{h}\right)$$

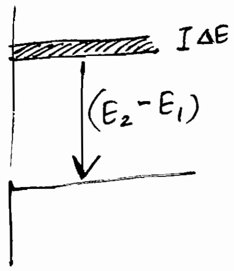
Usual case

But for resonant absorption γ should be $\left(\frac{\Delta E}{\hbar}\right)$ so that absorption occurs. Hence, I should have recoilless nucleus. It happens when ΔE is low $\simeq 10^{-4}$ eV

When ΔE is low, Δt (lifetime) is high. <u>Mossbauer case</u>

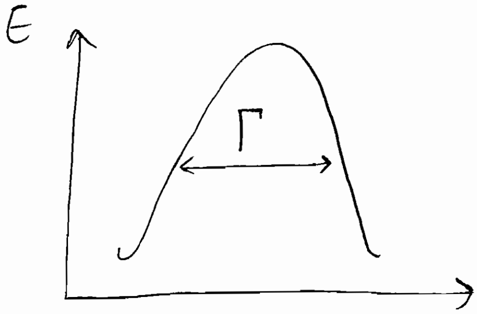
What is ΔE?

$\Delta E \, \Delta t \simeq \hbar$ ⊛ Order of $\hbar \simeq 10^{-15}$ eV sec



$\Gamma \Delta t \simeq 10^{-15}$ eV sec

$$T \simeq \Delta t \approx \frac{10^{-15} \text{ eV sec}}{10^{-4} \text{ eV}} \simeq \underline{10^{-11} \text{ sec}}$$



<u>Not every nucleus gives Mossbauer Effect i.e. every nucleus does not give gamma photons of particular frequency.</u>

$X \longrightarrow Y + \gamma$

✓ Also note γ's frequency has to be adjusted for doppler effect

Doppler Broadening $\Rightarrow \frac{\Delta \nu}{\nu} = \left(\frac{v}{c}\right) \Rightarrow \left(\frac{\Delta E}{E}\right) = \left(\frac{v}{c}\right)$

due to recoil of nucleus i.e. motion of source.

$$p_\gamma + p_r = 0 \Rightarrow p_\gamma = -p_r = -\left(\frac{E_r}{c}\right)$$

Note the $v_{recoil} \ll c \Rightarrow$ Non Relativistic measures can be applied $\Rightarrow \frac{p_\gamma^2}{2 M_{recoil}} = \boxed{\frac{E_r^2}{2 M_{recoil} c^2} = E_{recoil}}$

⊙ If whole of crystal recoils instead of single nucleus

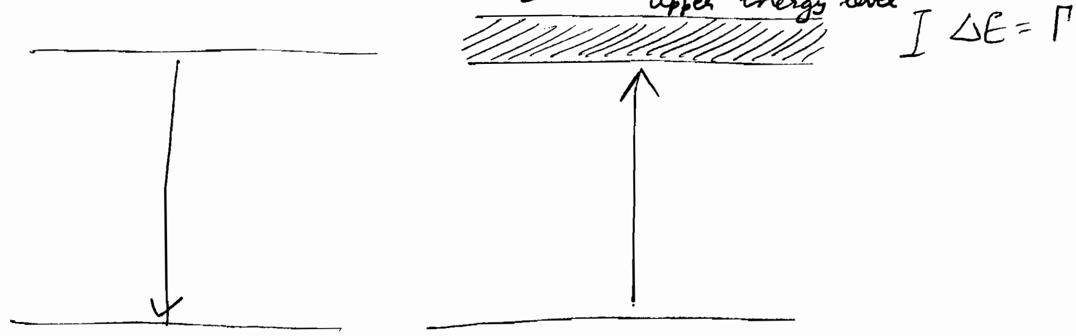
$\Rightarrow M_{recoil} \to \infty \Rightarrow E_{recoil} \to 0$ w/o reduction of E_r. Hence in such crystals, Mossbauer Effect is seen.

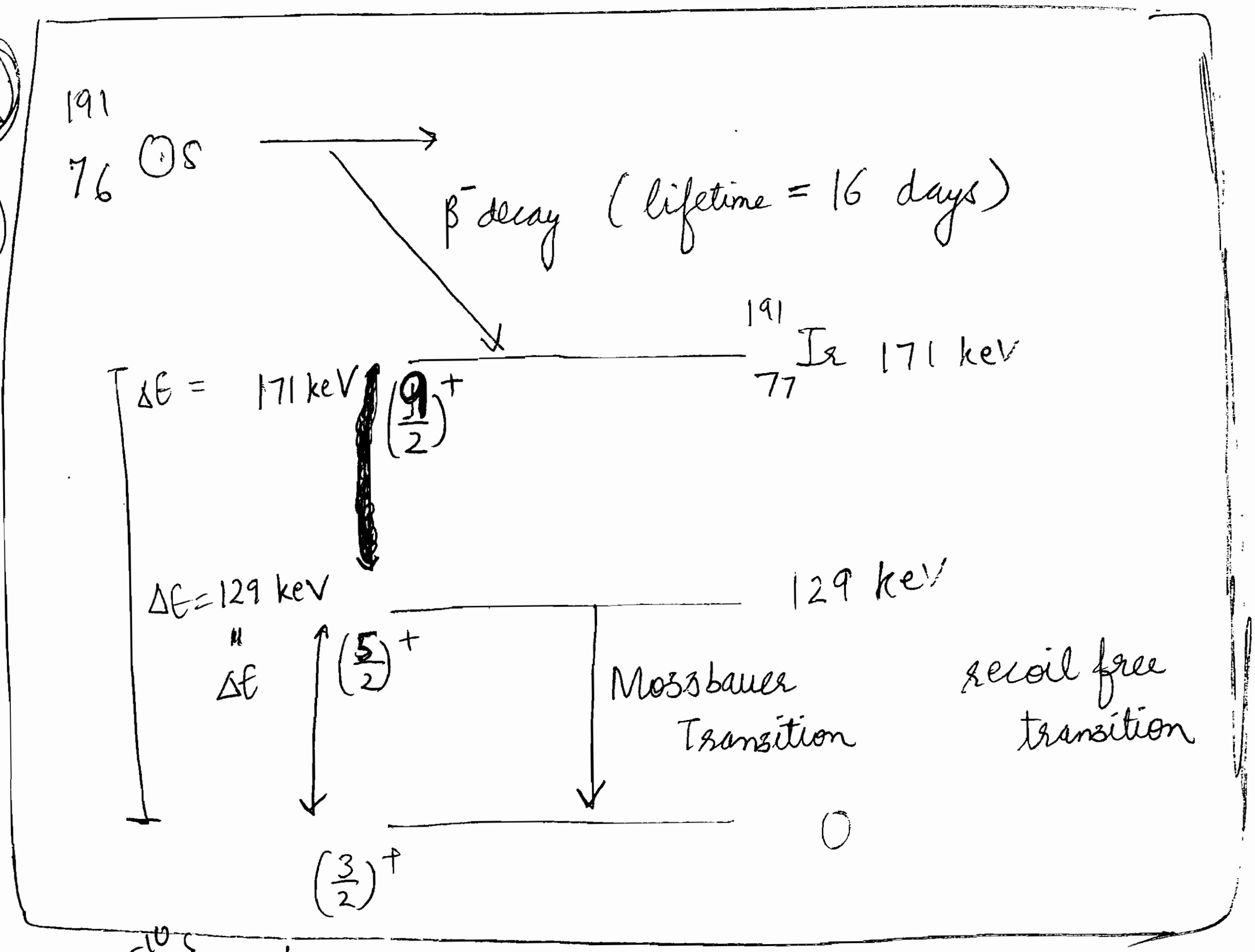
^{191}[Irridium] → 129 keV γ-photon's source

^{57}Fe

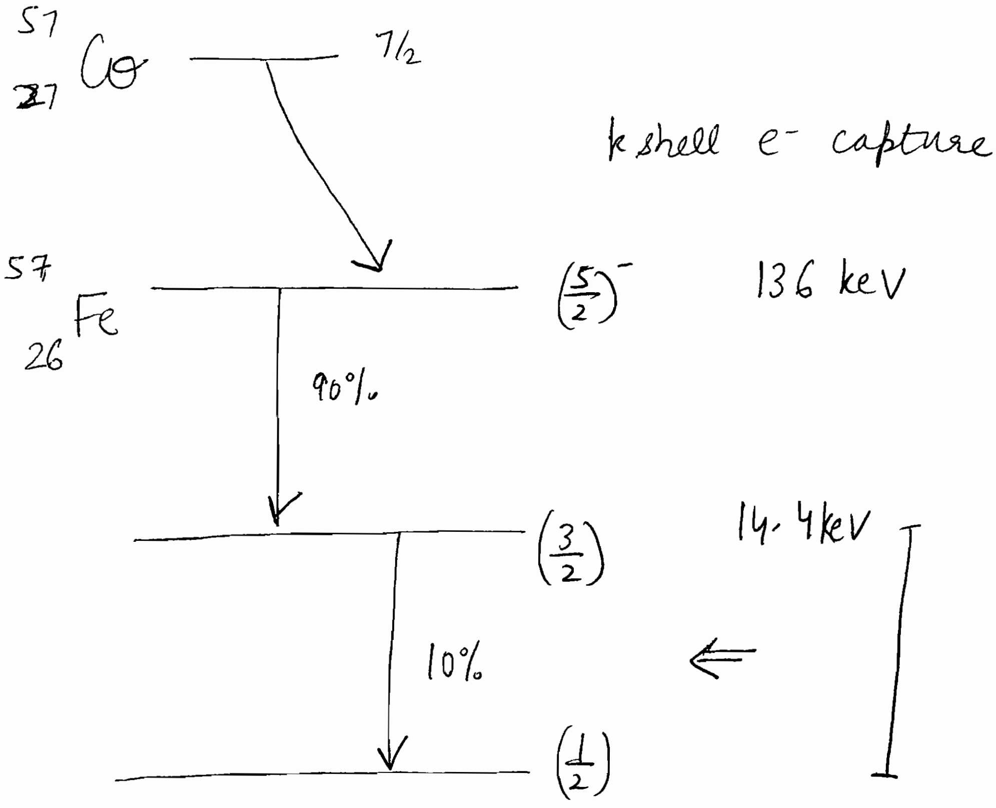
⊙ Note that for Mossbauer
(1) recoilless by embedding in crystal
(2) $\Delta E = \Gamma$ is small for the upper energy level

$E_{recoil} \to 0$ But $p = \sqrt{2mE} \not\to 0$





$\Delta t \simeq 10^{-10}$ s
$\Delta E \simeq 10^{-6}$ eV



- Massbauer is not given by any nucleus but a few like Ir, Fe, Ru, Ni

$\int$ *** RIN

Massbauer Effect

$E_r = \underline{14.4 \text{ keV}}$

$T = 1.4 \times 10^{-7}$ sec.

$\boxed{\Delta E = \frac{\hbar}{\Delta T}}$ $\Rightarrow$ width $= \left(\frac{10^{-15} \text{ eV sec}}{1.4 \times 10^{-7} \text{ sec}}\right)$

$= 4.6 \times 10^{-9}$ eV

It's a very sharp line ⟶

Massbauer Effect:

① Source of monochromatic γ photons.

② Experimental Confirmation of General Theory of Relativity

$$\frac{\Delta E}{E} = \left(\frac{v}{c}\right)$$

ΔE can be made due to gravitational energy

We calculate ΔE [redacted] $= \left(\frac{v}{c}\right) E$

ΔE matches with theoretical prediction.

(3) In chemical ~~shift~~ analysis of nuclei, (2I + 1) sublevels of I if Nucleus possesses dipole moment, in presence of ext. field. Its called hyperfine splitting.

It may may be

(a) Dipole splitting

(b) Quadru Pole splitting

(7B)

Nuclear Fission

To achieve stability. Their B.E per nucleon is high for heavier nuclei; hence nuclear fission occurs to stabilize. All such nuclei, prone to nuclear fission are

Fissile Nucleus

eg. $^{238}_{92}U$ — 99.3% } by weight

✓ $^{235}_{92}U$ — 0.7% }

✓ $^{239}_{92}U$

✓ $^{239}_{94}Pu$

✓ Fertile or fissionable material means that these are nuclei that can converted to fissile material

$$^{232}_{90}Th \longrightarrow [^{239}_{92}U]$$

$$^{238}_{92}U \xrightarrow{^{1}_{0}n} [^{239}_{92}U] \xrightarrow{\beta \text{ decay}} {}^{239}_{93}Np \xrightarrow{\beta \text{ decay}} {}^{239}_{94}Pu$$

- <u>Fuel : fissile material</u>
- <u>Raw Material : fertile material</u>

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow {}^{236}_{92}U$$

compound nucleus

$$\downarrow$$

$$^{98}_{40}Zr + {}^{136}_{52}Te$$

$$^{98}_{40}Zr \rightarrow {}^{98}_{41}Cb \rightarrow {}^{98}_{42}Mo$$

$$^{136}_{52}Te \rightarrow {}^{136}_{54}Xe$$

$$\Rightarrow \quad ^{235}_{92}U + ^{1}_{0}n \longrightarrow {}^{94}Sr + {}^{140}Xe + 2\,^{1}_{0}n$$

+210 MeV

Neutrons can be

1. Cold Neutron : lowest energy
2. Thermal Neutron : @ room temp.
3. Fast Neutron — upto MeV
4. Very Fast Neutron — upto 10 MeV
5. Ultra Fast Neutron — > 10 MeV

In the previous reaction, any neutron, even @ room temperature can be used to break it (^{235}U).
Hence spontaneous reaction.

$Q \simeq 210$ MeV for previous reaction

Every reaction requires certain activation energy.
Some bombardment is required for nuclear fission reactions
Its provided by neutrons. So it depends upon neutron whether it can provide certain amount of activation energy!!

→ Note that mass no. is conserved in nuclear reaction.
difference in ΔE comes due to difference in Binding Energies as $M_{nucleus} \neq \sum A$

Q. $U^{235} \rightarrow 200$ MeV

Find out annual requirement of U^{235} for having power output $= 1W$.

Ans/- Remember ratio: 99.3 : 0.7 ratio.

1 Watt : $1 \, J/sec = \frac{1}{(1.6 \times 10^{-19})} \, eV/sec$

$= 6.25 \times 10^{18} \, eV/sec$

$= 6.25 \times 10^{12} \, MeV/sec$

^{235}U required per sec.

$= \frac{6.25 \times 10^{12}}{200} = \underline{\underline{3.125 \times 10^{10} \text{ nuclei}}}$

Mass per sec $= 6.023 \times 10^{23} : 235$ gram

$\Rightarrow 3.125 \times 10^{10} \Rightarrow \frac{235}{6.023 \times 10^{23}} \times 3.125 \times 10^{10}$ gram

Mass of ^{238}U per sec. $\Rightarrow \frac{99.3}{0.7} \times \frac{235}{6.023 \times 10^{23}} \times 3.125 \times 10^{10}$ g.

How reactor is formed?

Component of reactions:

1. Fuel i.e. fissile material
2. Moderator i.e. 2.5 neutrons emitted on avg. per nuclear reaction ....
   We have to control chain reaction using moderators eg. Graphite
   Water
   Heavy water

Remember: 1st class if $m_1 = m_2 \Rightarrow$ velocities are interchanged

But this is not the only criteria. 2nd criteria is absorption cross section. For water, potential to stop neutron is maximum but its absorption cross section is small. Hence heavy water is used.

③ <u>Coolant</u> : liquid is circulated. It may be water or liquid gases.

PHWR

Boiling Water Reactor

liquid → steam → useful work

④ <u>Control Rods</u>.

Note that moderator was slowing down neutrons. Cadmium & Boron Rods are used to control the reaction

⑤ <u>Shield</u>

Thermal shield or Concrete shields

✓ <u>Fuel + Moderator</u> : mixed and formed a solution

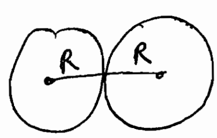
⇒ <u>homogenous plant</u>

if separate ⇒ <u>heterogenous plant</u>

Application of Semi Empirical Mass Formula in Fission

3 fines points to take care of.

$$ {}^{A}_{Z}X \longrightarrow \bigcirc \; {}^{A/2}_{Z/2} \;\; + \;\; \bigcirc \; {}^{A/2}_{Z/2} $$



① take $R_0 = 1.5$ fm

$$ E_c = \frac{\frac{1}{4\pi\epsilon_0}\left(\frac{Ze}{2}\right)^2}{2R_0\left(\frac{A}{2}\right)^{\frac{1}{3}}} $$

Coulomb Repulsion in critical case

$$ = \left(0.15\frac{Z^2}{A^{1/3}}\right) \text{MeV} \; ✓ $$

$${}^{A}_{Z}X \longrightarrow 2\, {}^{\frac{A}{2}}_{\frac{Z}{2}}Y + Q$$

Semi Empirical mass Formula

$$M(Z,A) = ZM_p + (A-Z)M_N - \left[a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm a_5, 0)\frac{1}{A^{3/4}}\right]$$

$$Q = \left[M(Z,A) - 2M\left(\frac{Z}{2}, \frac{A}{2}\right)\right] c^2$$

All fissile material are of odd A ⇒ as not there

$$Q = +a_2\left(A^{2/3} - 2A^{2/3}\left(\frac{1}{2}\right)^{\frac{2}{3}}\right)$$

$$+ a_3\left(\frac{Z^2}{A^{1/3}} - 2\frac{\left(\frac{Z}{2}\right)^2}{\left(\frac{A}{2}\right)^{\frac{1}{3}}}\right)$$

(3) we can take $Z(Z-1) \approx Z^2$

(2) $Q = |B.E._F| - |B.E._i|$

$$+ a_4\left[\frac{(A-2Z)^2}{A} - 2\frac{\left(\frac{A}{2}-Z\right)^2}{\left(\frac{A}{2}\right)}\right]$$ ⇐ this term cancels out

$$Q = \left(-3.42\, A^{\frac{2}{3}} + 0.22\frac{Z^2}{A^{\frac{1}{3}}}\right) \text{MeV}$$

For spontaneous fission $Q \geq 0$

$$\Rightarrow 0.22\frac{Z^2}{A^{1/3}} \geq 3.42\, A^{2/3}$$

$$\Rightarrow \left(\frac{Z^2}{A}\right) \geq 15$$

Fission Parameter for spontaneous fission

To better the result,

if $Q < E_{coulumb} \Rightarrow$ no reaction

$$Q = P.E. + K.E \text{ of daughters}$$

$\Rightarrow$ For reaction, $Q \geq E_{coulumb}$

$$\Rightarrow -3.42 A^{\frac{2}{3}} + 0.22 \frac{Z^2}{A^{\frac{1}{3}}} \geq 0.15 \frac{Z^2}{A^{1/3}}$$

$$\Rightarrow 3.42 A^{2/3} \leq 0.07 \frac{Z^2}{A^{1/3}}$$

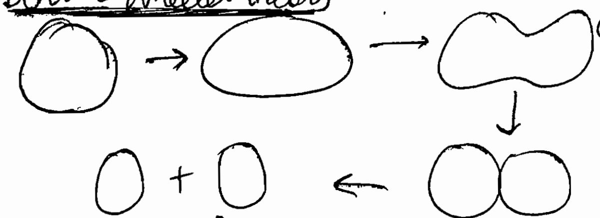
$$\Rightarrow \boxed{\frac{Z^2}{A} \geq 49}$$

$$\boxed{\frac{Z^2}{A} \geq 43}$$

Bohr and Wheeler's Approximation using liquid drop model and using legendre's polynomial

$$R = R_n P_n(\cos\theta)$$

Bohr & Wheeler theory



Via Quantum Mechanical Tunneling, Fission can take place even with $(Q - E_A) < E_{coulumb}$

↑ Activation energy

→ Experimental determination of $\mu_{nucleus}$

Ex " " " μ_{proton}, $\mu_{neutron}$ via magnetic Resonance.

via Raabi

[neutron : unstable
proton : stable]

→ Neutron is covered by negative charge cloud. Unsymmetric charge distribution in neutron.

→ Absorption Cross Section } 4 factors determining Probability of Nuclear Fission

→ Bohr Wheeler Condition

→ Internal Conversion from book

$[\text{Prob}(\alpha)] \propto [Z^3]$

→ Mossbauer : Width of nuclear energy levels is extremely narrow, then only resonant absorption.

→ Unification Theory : in Energy Scale

⊛ Remember $\Delta E\, \Delta t \simeq \hbar$ for pi meson. Enough energy must be supplied to a nucleon so that its emission conserves energy. Thus at least $m_\pi c^2 \simeq 140$ MeV is required. This amount of energy is produced in LHC, and therefore such particles could be studied.

~~Solid State Physics~~

Standard Model

Present model of elementary particles and their interaction. In the model, the strong, weak and electromagnetic forces appear as different manifestations of one basic phenomenon, with leptons and quarks finding natural places within the scheme. By placing leptons & quarks in the same framework, it becomes possible to explain why the electrons (leptons) and the protons (composition of quarks) have electric charge of exactly the same magnitude.

Particles

① Spin ½ fermions : 3 generation of leptons
3 generation of quarks
All their antiparticles

② Spin 1 Gauge Boson (12 in number) : 1 massless electroweak boson γ
3 massive electroweak boson $W^{\pm}, Z^{0}$
8 coloured gluons

③ Spin-0 Higgs Boson

Interaction EM coupling of γ
Weak interaction of $W^{\pm}, Z^{0}$
Strong Interaction gluons

Higgs Boson

Higgs mechanism is responsible for giving mass to gauge bosons. [redacted] Higgs mechanism is the mechanism by which the existence of a spin 0 particle can give a gauge boson mass w/o breaking the gauge symmetry. Higgs showed that a field, called the Higgs field, must exist everywhere in space. By interacting with Higgs field, particles acquire their characteristic mass. HIGGS BOSON mediates the action of Higgs field. Mass of Higgs Boson could be 1 TeV/c^2

Grand Unification Theory

According to GUT, the particles should belong to a group representation. The simplest possible GUT is labelled SU(5).

Intro

Success of electro-weak unification has prompted physicists to pursue vigorously the attempts at unifying the strong nuclear and electroweak interactions. Such grand unification will do away with the distinction between quark and leptons at sufficiently high energies. All the three interactions are then expected to be manifestations of different facets of single universal gauge force. Such a possibility arises due to the same value of the spin for gauge boson in the 3 cases. Unification Energy is quite large. After universe began expanding at a time when its thermal energy was comparable to unification energy, unification effects were large. Now the effects are extremely low bcoz thermal energy of universe is low. Their observed vastly different strengths at low energy result from differing normalizations due to successive spontaneous symmetry breaking.

Consequence

A startling consequence of the theory entail the disappearance of distinction between quarks & leptons, which gives rise to possibility of transformation of protons into leptons and pions. The protons would then not be stable particles.

2nd important consequence of GUT is the existence of very massive magnetic monopoles.

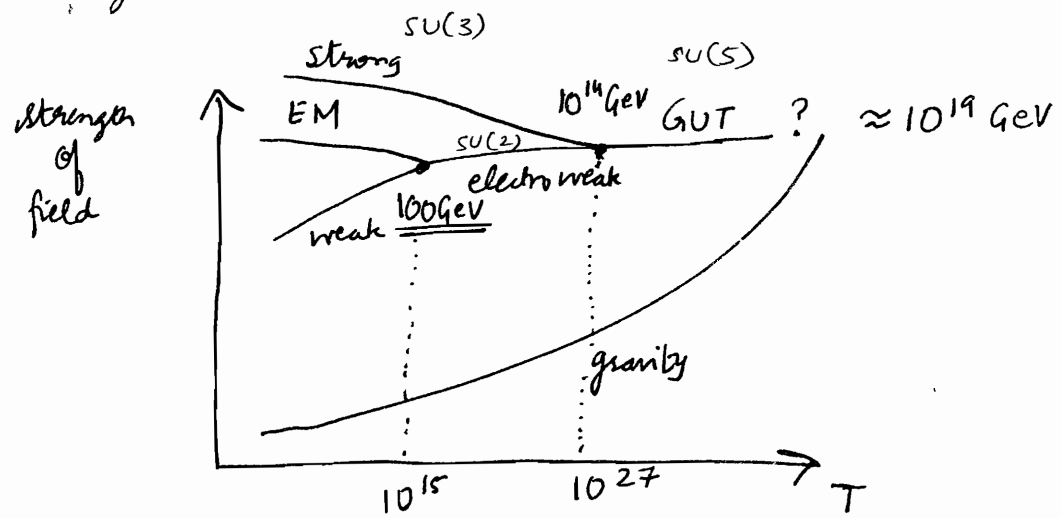
3rd it will provide solid support for big bang theory of creation of universe. Theories of smallest constituents of observable matter are thus seen to have profound implications on cosmology.

Theory

SU(5) representation has a place for all the fermion of one generation and will treat the quarks and leptons on equal footing. The interaction is with gauge bosons associated with an SU(5) local gauge invariance : that means 24 gauge bosons. 12 gauge bosons have already been ~~discovered~~ collected which are γ, $W^{\pm}$, Z^{0}, 8 gluons. Group theory says that six others are each coloured, one of red, green or blue with electric charge

$$-\frac{1}{3}|e| \; (Y_R, Y_G, Y_B) \quad \text{or} \quad -\frac{4}{3}|e| \; (X_R, X_G, X_B)$$

Remaining 6 are antiparticles of these six.
They are called "lepto quark or diquark bosons".
Thus lepto quark are ~~hypothetical~~ particles that carry information between quarks & leptons given a generation and allowing quarks and leptons to interact.
They are colour triplet bosons that carry both lepton and Baryon number.



use : in π meson : $\Delta t = \left(\frac{1.4 \text{ fm}}{c}\right)$

in fission parameter : $R = 1.5 A^{1/3}$ fm

Mass of π meson

$$\Delta t = \left(\frac{1.4\ fm}{c}\right)$$

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{6.6 \times 10^{-34}}{2\pi\ \Delta t} = \underline{140\ MeV}$$

$$= \underline{\underline{280\ \text{times}\ m_e}}$$

Experimentally, $\underline{\underline{E = 270\ m_e}}$

Process of exchange

Protons and neutrons are both supposed to be surrounded by a virtual cloud of mesons.

The transfer of π^+ meson from proton cloud to neutron cloud results only in a change of identity of the particle i.e. in the final stage we have, again, a proton and a neutron with same energy as in initial stage.

$$p + n = n' + \pi^+ + n = n' + \pi^+ + n \rightarrow n' + p'$$

Similarly $p + p = p' + \pi^0 + p = p' + p'$

$$n + n = n' + \pi^0 + n = n' + n'$$

$$n + p = p' + \pi^- + p = p' + n'$$

Thus the force field between 2 protons or between 2 neutrons is carried by π^0 mesons. But the force field between a proton and a neutron can be carried by a charged π^+ or π^- meson.

(*) Force field को एक जगह से दूसरी जगह carry करने के लिए एक Particle चाहिए होता है, उसे Exchange Particle बोलते हैं !!

Difference = 0.8 MeV is due to electrostatic forces of repulsion. If allowance is made for this force, then (n-n) and (p-p) forces are equal.

(5) Strongest Forces in nature. 10^{40} times gravitational forces.

(6) Spin Dependent

It has been observed that force of attraction between two nucleons of parallel spin (↑↑) is stronger than the force between two nucleons having anti parallel spin (↑↓)

(7) Non Central

(8) Exchange Forces (explained below)

(9) At too close distance of approach between the nucleons, nuclear forces become force of repulsion.

Meson Theory of Exchange Force by Yukawa

Just as photon is quantum of electromagnetic field, meson is a quantum of nuclear field. Meson field is of extreme short range b'coz the exchange of mesons between the nucleons can take place only when nucleons are close together, otherwise principle of conservation of energy will be violated b'coz a meson has a finite rest mass and energy will have to be supplied to remove it away from a nucleon.

Remember $\Delta E \, \Delta t \approx \hbar$

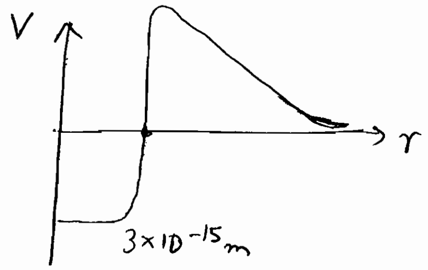
Yukawa predicted the existence of meson. It was finally discovered in 1947.

The Quantity $\left(\frac{Z^2}{A}\right)$ is called Fission Parameter.

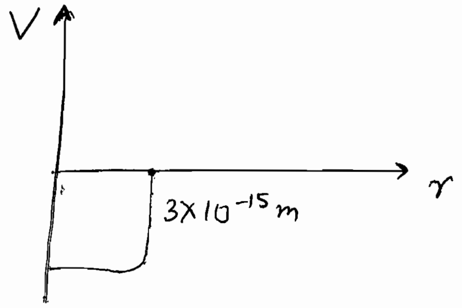
For fission to take place under action of neutron $\frac{Z^2}{A}$ should be more than 49. Bohr and Wheeler approximated this parameter as 43.

Nuclear Force Features

① **p-p force** : determined from proton - proton scattering



② **n-p force** : determined from neutron - proton scattering



③ **Short Range Force and Saturation Property**

Proof : If long range force ⇒ each nucleon interacts with every other nucleon ⇒ Binding Energy is proportional to $^AC_2$ i.e. A^2 ⇒ Binding Energy per nucleon $\propto A$

But Binding Energy is almost const.

Hence not a long range force.

④ **Charge Independent**

Proof : From study of mirror nuclei, we find that 3 types of forces (p-p), (p-n) and (n-n) are almost of equal magnitude. eg. ^3_1H and ^3_2H

^3_1H 8.5 MeV ⇔ 2 (p-n) + 1 (n-n)

^3_2He 7.7 MeV ⇔ 2(n-p) + 1 (p-p)

Fission

Since the heavy nuclei have large (N/Z) ratio, the fragments contain an excess of neutrons. This excess is reduced by emission of neutrons by the fragments as soon as they are produced. These are called Prompt Neutrons. The fragments reach stability by subsequent β decays.

Fission Reaction : $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U$

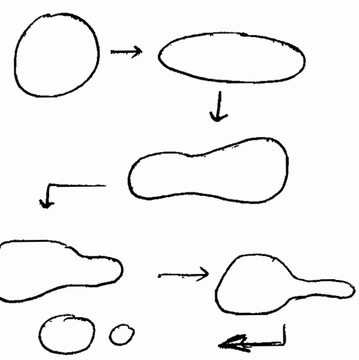
$$^{140}_{54}Xe + ^{94}_{38}Sr + ^{1}_{0}n + ^{1}_{0}n \longleftarrow$$ Unstable

Explanation from Liquid Model

Amount of energy that has to be spent to make the fission possible is called Activation Energy. In the liquid drop model, just as the liquid drop is most stable in spherical shape, nucleus has greatest stability if sum of surface energy and coulumb energy is minimum.

When neutron is captured by nucleus, its deformed into an ellipsoid. Surface Area increase and electrostatic energy reduces due to increase in mean distance. Condition for stability is no longer there and nucleus starts oscillating by stretching and contracting.

At excitation energies of nucleus less than activation energy, deformation does not reach critical stage & nucleus returns to basic energy after emitting γ photon.



From (λ) and (μ), we need to eliminate θ,

$$\therefore \quad p_y^2 = p_b^2 \sin^2\phi + (p_a - p_b \cos\phi)^2 = p_b^2 + p_a^2 - 2 p_a p_b \cos\phi$$

$$\Rightarrow 2 m_y E_y = 2 m_B E_B + 2 m_A E_a - 2 \cdot 2 \cdot \sqrt{m_B m_A E_a E_B} \cos\phi$$

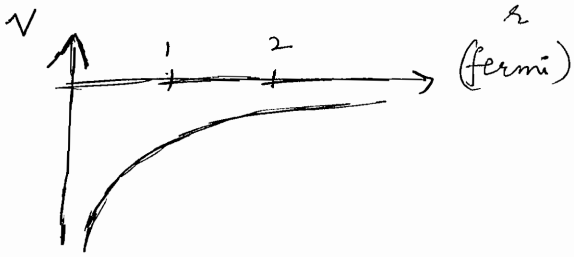
$$\Rightarrow E_y = \frac{m_B}{m_y} E_b + \frac{m_A}{m_y} E_a - \frac{2\sqrt{m_a m_b E_a E_b} \cos\phi}{M_y}$$

⇒ From (1),

$$Q \text{ value} = \left(\frac{m_B + m_y}{m_y}\right) E_b + \left(\frac{m_A - m_y}{m_y}\right) E_a - \frac{2\sqrt{m_a m_b E_a E_b}}{M_y}$$

⊙ <u>Yukawa's Potential</u> :

$$⊙ \quad V(r) = \frac{-g^2}{4\pi r} e^{-\left(\frac{r}{r_0}\right)}$$



where $r_0 = \left(\frac{\hbar}{m_\pi c}\right)$: m_π : mass of π meson

& $g^2 \simeq 15\, \hbar c$

○ Yukawa's Potential is used to explain nucleon - nucleon interaction at higher values of r

⊙ Hafnium (Hf)
Tantalum (Ta) } Nuclear Isomers used in weaponry

 (हफ्ता)

Q value of a reaction

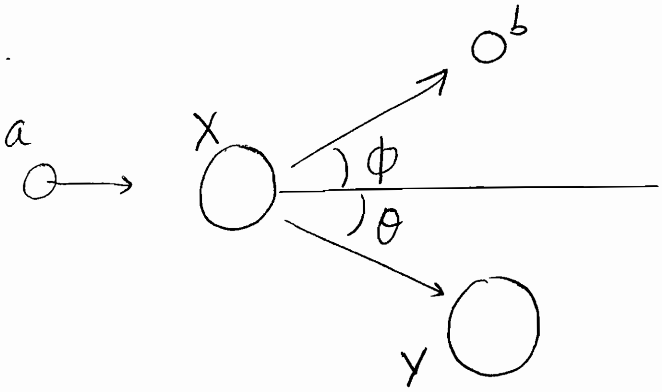
$$a + X \longrightarrow Y + b$$ [redacted]

$$Q = (E_Y + E_b) - E_a \quad — (1) \qquad (X \text{ at rest})$$

[where E_i represents kinetic energy of i^{th} particle]

To calculate Q, we need E_Y which is difficult to measure. Note that E_a and E_b are known.

To calculate E_Y, let us use the conservation laws,



Momentum Conservation:

$$p_b \cos\phi + p_Y \cos\theta = p_a \quad — (\lambda)$$

$$p_b \sin\phi = p_Y \sin\theta \quad — (\mu)$$

Energy conservation:

$$m'_a c^2 + m'_X c^2 = m'_b c^2 + m'_Y c^2$$

$$\Rightarrow (m_a c^2 + E_a) + (m_X c^2) = (m_b c^2 + E_b) + (m_Y c^2 + E_Y)$$

(Note that we cannot use this equation for E_Y as rest masses are unknown)

$$\Rightarrow Q = (E_Y + E_b) - E_a = (m_a + m_X) c^2 - (m_b + m_Y) c^2 \quad — (2)$$

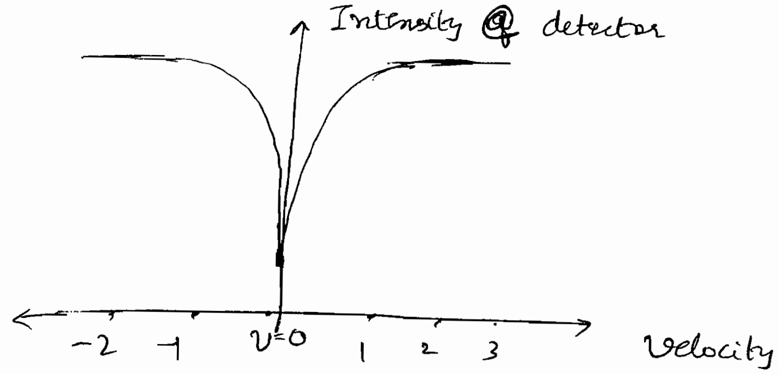
[Hence we have two expressions for Q value]

no use in this ques.

The source and absorber nuclei are placed in two cryostats maintained at a low temperature. Cooling reduced the thermal vibrations and accompanied changes in Energy. NO resonance would normally be expected. But due to Mossbauer effect, resonance would take place.

Dobbler effect could be studied from the rotating mechanism shown in the figure.

$$\frac{\Delta E}{E_\gamma} = \left(\frac{v}{c}\right) \quad \text{from doppler ;}$$



Application of Mossbauer Effect

① Half width of only about 10^{-6} eV makes this absorption on extremely sensitive test of any influence which would shift the frequency. It is sensitive enough to measure the Zeeman Splitting, from magnetic field, of the nucleus. Such processes have been used to test the gravitational red shift of general relativity.

② Verification of doppler effect $\frac{\Delta E}{E} = \left(\frac{v}{c}\right)$

③ Source of monochromatic γ photons.

④ Transformation of iron catalysts and enzymes are studied using Mossbauer phenomenon showed by ^{57}Fe

Mossbauer Effect

Under certain circumstances, γ-rays could be emitted from nuclei without any loss of energy due to recoil of the nucleus. As such, these γ-rays have same energy as the transition energy between the two states. This type of transition is known as Recoilless Transition and the effect is called Mossbauer Effect. The experimental set up to observe the effect is called Mossbauer Spectroscopy.

Mossbauer carried out his experiment using 129 keV γ-rays from ^{191}Ir produced in β-decay of ^{191}Os.

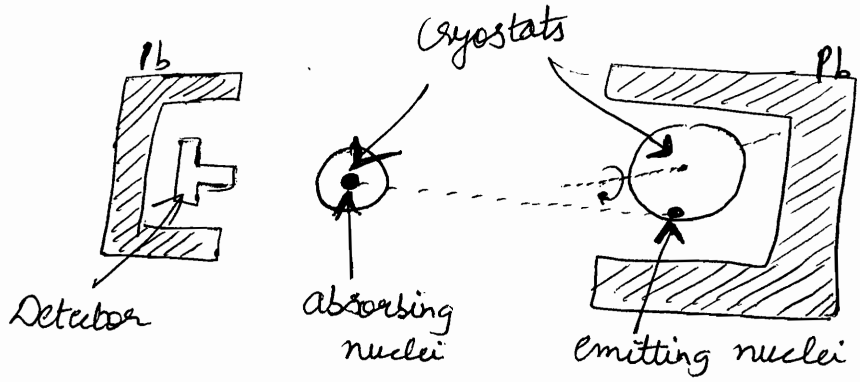
← diagram

129 keV excited level in ^{191}Ir has a mean life of 1.3×10^{-10} s, so that it was width $\Delta E = \frac{\hbar}{\tau} \approx 10^{-6}$ eV

Hence highly monochromatic ray.

In a recoilless transition, the emitted γ-ray is highly monochromatic and hence can induce resonance transition in another identical nucleus, provided the absorption process is also recoilless.

Mossbauer produced recoilless transition by embedding the emitting nuclei in a solid lattice of atoms.



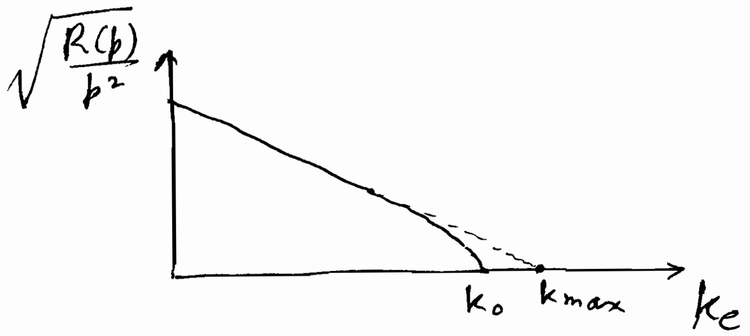
⊙ Curie's Plot (to determine mass of neutrino)

Consider the reaction $n \rightarrow p + e^- + \bar{\nu}$

Kuries plot is the graph between $\sqrt{\frac{R(p)}{p^2}}$ and ke

where p, k_e : momentum & K.E. of electron

$R(p)$: rate of emission of e^-, with momentum p, per unit time per unit momentum.



kurie plot is straight line, who x-intercept on intercept (k_o) differs from extrapolated intercept (k_{max}) due to mass of neutrino

$$M_{\bar{\nu}} = \left(\frac{k_{max} - k_o}{c^2}\right)$$: Limit on the mass of neutrino

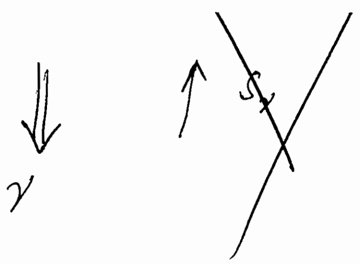
⊙ 1 Curie = 3.7×10^{10} decay/second

(unit of radioactivity decay rate)

i.e. unit of $\left(\frac{dN}{dt}\right)$

$$\left(\frac{dN}{dt}\right) = \lambda N = \lambda N_0 e^{-\lambda t}$$

1 Bacquerel = 1 Bq = 1 decay/second



(★) Isotopic spin number I is conserved in strong but NOT in weak or EM interaction.

But z-component of I i.e. I_3 is conserved for strong and EM reactions but NOT conserved in weak.

Thus it should be noted that for EM interaction, I_3 is conserved but I is not.

$$\pi^0 \longrightarrow \gamma + \gamma$$

I:	1	0	0	(not conserved)
I_3:	0	0	0	(conserved)

Hence EM reaction [gamma ray]

$$K^0 \longrightarrow \pi^0 + \pi^0$$

I:	$\vec{\frac{1}{2}}$	$\vec{1}$	$\vec{1}$ $= 0, 1, 2$.	(not conserved)
I_3:	$-\frac{1}{2}$	0	0	(not conserved)

Hence weak interaction. [decay of strange particle]

(★) if $\Delta S \neq 0 \Rightarrow$ weak interaction
but
$\Delta S = 0 \not\Rightarrow$ strong / EM

Now we have to check
I_3, I, Y

if all conserve, then we can say strong / EM

eg. $n \longrightarrow p + e^- + \bar{\nu}$

$\Delta S = 0$ $\Rightarrow$ weak
$\Delta I_3 \neq 0$

$-\frac{1}{2}$ $\frac{1}{2}$ 0 0

★ <u>Muon</u> (lepton)

Its a lepton. $\begin{bmatrix} Spin = \frac{1}{2} \\ B = 0 \\ Q = -1 \\ S = 0 \end{bmatrix}$ (def^n for any particle)

<u>Production from π mesons</u>

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$
$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

Majority of the cosmic particles at secondary level are muons.

<u>Decay into e^- and e^+</u>

$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$
$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

★ Proton is a stable baryon.
Neutron in free space decays with mean life time of 1·5 minutes

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

Also electron, neutrinos are stable.
e^- $\nu_e\ \nu_\mu\ \nu_\tau$
Hence no lifetime.

★ NO quantum is assigned to the meson group of particles. This means that any no. of mesons can appear on R.H.S. of a possible reaction or decay even when there are no mesons on the left side of the equation

Radioactivity

activity $= \left(\frac{dN}{dt}\right) = -\lambda N$

$\Rightarrow N = N_0 e^{-\lambda t}$

$$N = N_0 e^{-\lambda t}$$

Half life $\tau \Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda \tau}$

$$\Rightarrow \frac{1}{\lambda} \ln 2 = \tau$$

Mean life $\bar{T} = \left(\frac{1}{\lambda}\right) = \left(\frac{\tau}{\ln 2}\right)$

α Particle energy

$$p_y = p_\alpha \quad — (1)$$

$$Q = E_y + E_\alpha \quad — (2)$$

$$\Rightarrow Q = \frac{p_\alpha^2}{2M_y} + \frac{p_\alpha^2}{2M_\alpha} = \frac{p_\alpha^2}{2M_\alpha}\left[1 + \frac{M_\alpha}{M_y}\right]$$

$$\Rightarrow k.E_\alpha = Q\left[\frac{M_y}{M_y + M_\alpha}\right] = Q.\frac{(A-4)}{A} = \left(\frac{A-4}{A}\right) Q$$

β-decay

- β decay:
  - β^- — Beta Decay
  - β^+ — Positron Emission
  - e^- capture — Electron Capture

- Neutrino interact with matter only through the process of inverse β-decay.

- Whereas a neutron outside a nucleus undergoes negative β decay into a proton (half life $\approx$ 10 min) b'cos its mass is greater than that of proton, the lighter proton cannot be transformed into neutron except within nucleus

⊙ Electron capture is competitive with positron emission, since both processes lead to the same nuclear transformation. Electron Capture occurs more often than positron emission in heavy nuclides b'coz the electrons in such nuclides are relatively close to the nucleus, with promotes their interaction with it.

Inverse β decay

$$p + \bar{\nu} \longrightarrow n + e^+$$

$$n + \nu \longrightarrow p + e^-$$

Inverse Beta decay have extremely low probabilities which is why neutrinos and anti neutrinos are able to pass through such vast amounts of matter.

Reines and Cowan proved this reaction.

Sufficient flux of antineutrino available near a nuclear reactor. Tank of water containing Cadmium compound in solution provided protons. Surrounding the tank was Gamma Ray detectors.

Immediately after a proton absorbed a neutrino, the created positron encountered an e- and both were annihilates. Gamma Ray detectors responded to resulting pair of gamma photons (0.51 MeV)

$$\underset{(0.51)}{e^+} + \underset{(0.51)}{e^-} \longrightarrow \underset{(0.51)}{\gamma} + \underset{(0.51)}{\gamma} \quad \text{MeV}$$

Meanwhile the neutron created was captured by Cadmium Nucleus. The new heavier Cadmium nucleus then released about 8 MeV of excitation energy divided among 3 or 4 photons, detected later by Detector.

(*) Slow neutrons have more absorption cross section and therefore moderators in nuclear reactors to slow down neutrons to have better reaction.

(*) $^{14}N(\alpha, p)^{17}O \Leftrightarrow {}^{14}N + {}^{4}\alpha \longrightarrow {}^{1}p + {}^{17}O$

· Note that when symbols of elements are written, writing Z is redundant.

$$Z \Leftrightarrow \text{Symbol} \quad \text{eg} \quad N \Rightarrow Z = 7$$

(*) Activation energy is required for fission reaction. It can be provided by Bombarding a neutron or Gamma Ray exposure or Proton Bombardment. Some nuclides are so unstable as to be capable of spontaneous fission, but they are more likely to undergo α-decay before this takes place.

(*) <u>Stages of Chain Reaction</u>

The condition for a chain reaction to occur in an assembly of fissionable material is that at least one neutron produced in a fission must, on the average, cause another fission.

If too few neutrons cause fission, reaction will slow down & stop. Its called SUBCRITICAL STAGE.

If precisely one neutron per fission causes another fission, energy will be released at a constant rate. It occurs in nuclear reactor and is called CRITICAL STAGE.

If the frequency of fission increases, energy release will be so rapid that an explosion will occur. It occurs in Atomic Bomb and is called SUPERCRITICAL STAGE.

(*) Light water (H_2O) is an efficient moderator but not used with natural uranium. Why? Protons tend to capture neutrons to form deuterons in the reaction $^1H(n,\gamma)^2H$. light water reactors, therefore, cannot use natural Uranium for fuel but need enriched Uranium whose ^{235}U content has been increased to about 3%. (normally 0.7%)

(*) In reactors,

→ Control rods of Cadmium or Boron which are good absorbers of slow neutrons, can be slid in and out of the reactor core to adjust rate of chain reaction.

→ water circulating around fuel is kept at high Pressure to prevent boiling.

(*) In star interior elements upto ^{209}Bi can be formed by fusion. Supernovas are responsible for heavier element creation..

(★) For a weak interaction, $\Delta S = 0$ or ± 1 and nothing else

eg. $n \rightarrow p + e^- + \bar{\nu}$ $\quad \Delta S = 0$

↑ <strangeness>

eg. $\Xi^- \longrightarrow n + \pi^-$

S $\quad -2 \quad 0 \quad 0$

$\Delta S = 2$ Hence not permitted

Hence the reaction occurs in two steps:

$\Xi^- \longrightarrow \Lambda^0 + \pi^-$

S $\quad -2 \quad -1 \quad 0$ $\quad \Delta S = +1$

$\Lambda^0 \longrightarrow n + \pi^0$

S $\quad -1 \quad 0 \quad 0$ $\quad \Delta S = +1$

$\Xi^- \longrightarrow n + \pi^- + \pi^0$

(★) ✓ Electromagnetic interaction also affects uncharged particles that have electric or magnetic moment.

(★) Coupling constant is a dimensionless (no E dimensions) constant (no 'r' term)

✓ For EM: $g_{em} = \alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = \frac{1}{137}$

✓ For Gravitational $g_m = \frac{Gm^2}{\hbar c} = 6 \times 10^{-39} \approx 10^{-40}$

g is measure of strength of interaction.

e : 1.6×10^{-19} C
m : 1.66×10^{-27} kg

(*) $W^{\pm}$ responsible for normal Beta decay.
Z : neutral and heavier than W.

(*) Strong Forces are extremely fast. Characteristic time of interaction ≈ 10^{-23} seconds.

while

Weak Forces are comparatively slower. Characteristic time of interaction ≈ 10^{-8} seconds.

(*) All Quarks have spin $\left(\frac{1}{2}\right)$ and even parity

Initially usd ($) were discovered. later on to explain exception, other quarks were discovered.

(*) <u>Colour</u>

Presence of 2 u-quarks in proton and 3 s quarks in Ω^- hyperon violated Pauli's Exclusion principle which must be obeyed by quarks as they are half spin fermions. To resolve this difficulty, it was suggested that quarks & antiquarks have an additional property called colour which had 3 possibilities : red, blue and green for quarks and 3 possibilities anti-red, anti blue and anti-green for antiquarks.

. According to colour hypothesis all the three quarks in a Baryon have different colours which satisfies Pauli Exclusion Principle since all of them are in different states even if two or three are otherwise identical. Such a combination is considered to be colourless. by analogy with the way in which green, blue & red light combine to make white light.

According to colour hypothesis, meson is supposed to be consisting of quarks of one colour and an antiquark of corresponding anti colour, thus cancelling the

the colour effect as it is supposed that a colour & its anticolour combine to form white.

Thus we find that the Hadrons are always colourless. Quark colour is a property that has significance only within the hadrons but is never directly observable.

Importance of Colour

Gluons are massless, spin 1 particles travelling at the speed of light and each one carries a colour and an anti colour. [Choose 1 colour in 3 ways, 1 anticolour in 2 ways ⇒ 6 gluons]

- ✓ Blue Quarks emits blue - anti-red - gluon and becomes red quark.
- ✓ Red quark that absorbs blue - anti-red gluon becomes blue quark.

Thus the field that binds the quarks is a colour field. This study is called Quantum Chromodynamics. Colour plays the same role in strong interaction that charge plays in electromagnetic interaction.

(★) Mesons hold together nucleons to form nuclei
and
Gluons hold togther quarks to form nucleons

(★) Dirac's Theory of Antiparticle

From Dirac's Theory, an e^- can have negative as well as positive energies.

(Negative energy वाला सभी e^- होते हैं !! absence of negative energy is Positron)

$$E = \pm\sqrt{p^2c^2 + m_0^2c^4}$$

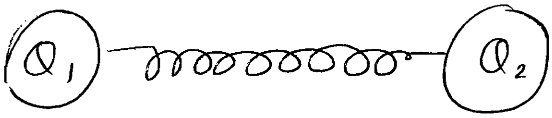
Dirac suggested that all negative energy states are normally filled. Pauli Exclusion Principle then prevents any other e^- from dropping into negative states. But if an e^- in the the sea of filled negative states is give energy more than $2m_0c^2$, then it can jump out of this sea and become an electron with positive energy. Process leaves behind a hole in the negative energy e^- sea, just like a hole in a semiconductor energy band, behaving as if it is a particle of positive charge → a positron. The result is materialization of photon into an $e^- - e^+$ pair.

$$\gamma \rightarrow e^+ + e^-$$

Flavor

Flavor refers to other types of quarks apart from u, d and s trio. They include charm, top and bottom.

Reasoning for Colour/Quark Confinement



Two Quark are proposed to be connected by a hypothetical spring such that attractive force b/w quarks moves up as quarks move apart from their normal spacing. This means more and more energy is required for their separation.

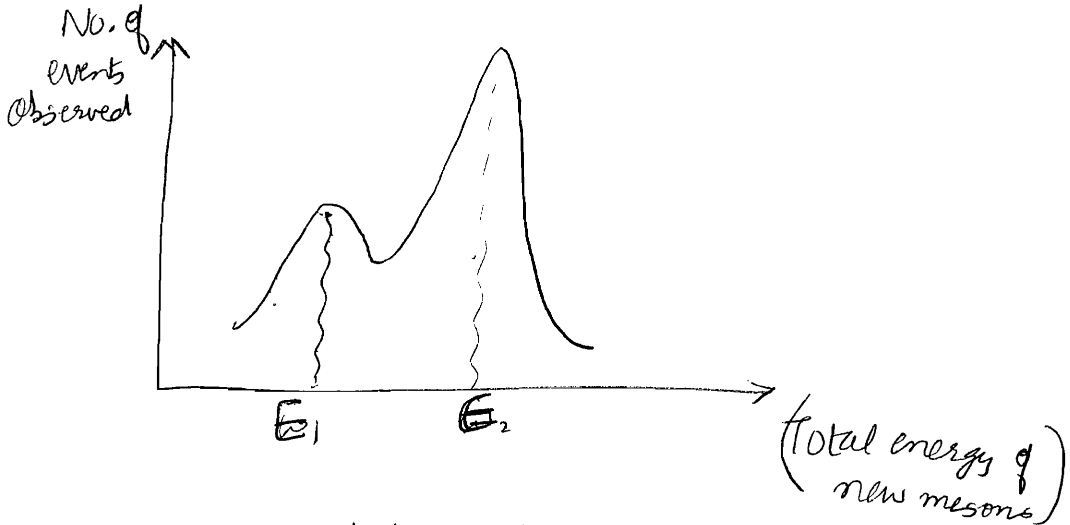
But with enough energy added, instead of a quark breaking free from the others in a hadron, excess energy goes into producing quark-antiquark pair which results into meson production that escapes.

(*) Graviton has spin = 2, and is massless.
Higgs Boson has spin = 0. But its very heavy : 10^6 $\frac{MeV}{c^2}$ or $1 \frac{TeV}{c^2}$

Resonance Particles

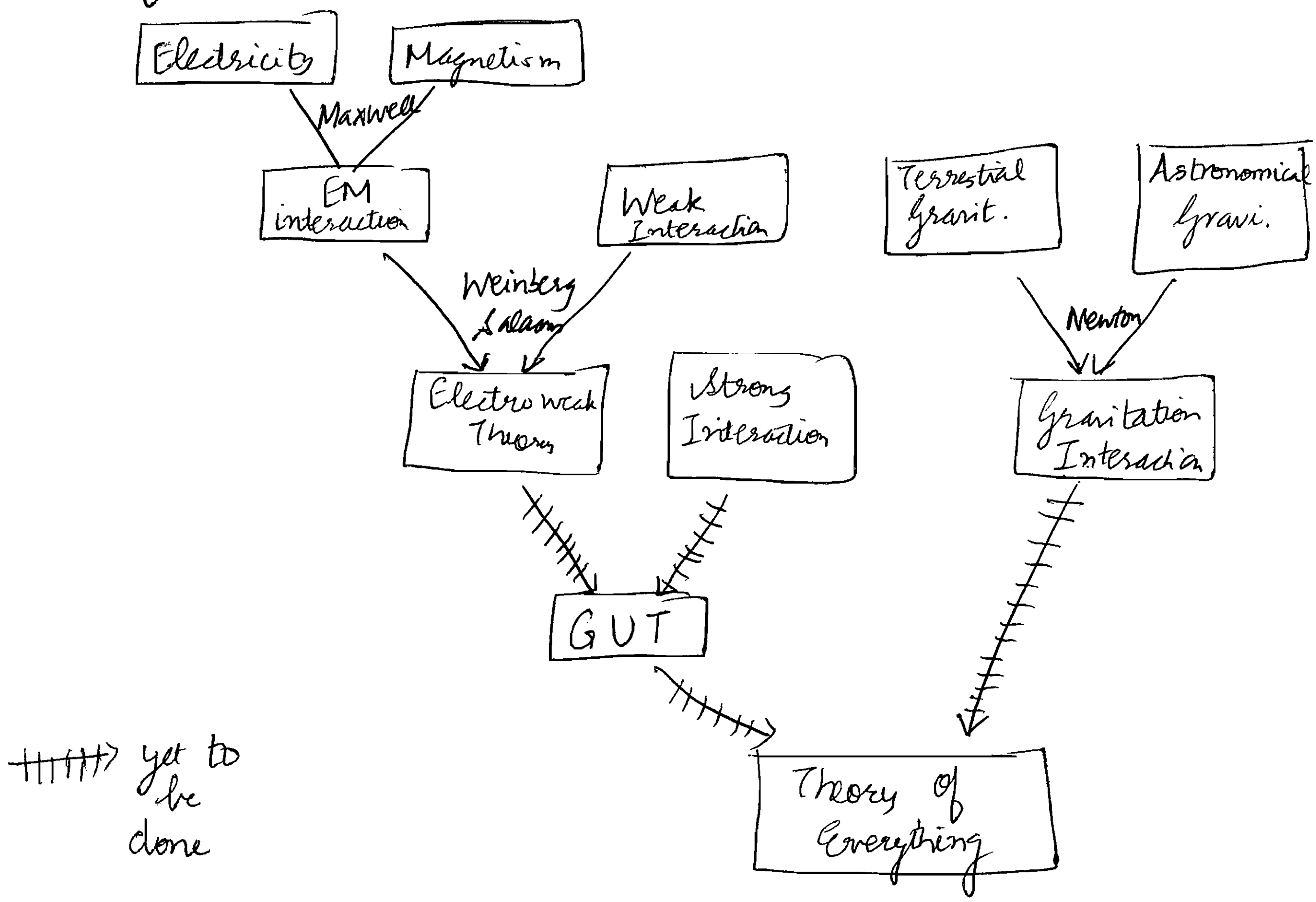
Experimental evidences suggest existence of many hadrons with lifetimes $\approx 10^{-26}$ sec. Such particles cannot be detected by recording their creation & subsequent decay, as the distance they travel even at c $\approx 3 \times 10^{-15}$ m. Instead such particles appear as resonant states in the interaction of longer lived hadrons. Resonant particles are equivalent to energy levels in atoms.

Consider $\pi^+ + p \longrightarrow \pi^+ + p + \pi^+ + \pi^- + \pi^0$



The resonant states observed in the experiment suggest the presence of resonant particles that are extremely short live and hence not observed.

✪ Unification



⊛ Photon vs Pion

	Photon	Pion
①	Mediating particle for EM interaction	Mediating particle for strong nuclear interaction
②	Photon has spin = 1	Pion has spin = 0
③	Neutral	+1, -1, 0 : 3 states
④	Stable	Unstable

Electroweak Theory

Connection between weak and EM interaction was independently developed in 1960s by Salaam and Weinberg. The key problem in constructing a theory was that carriers of weak forces have masses while carriers of EM force namely photons are massless.

They showed that at certain primitive levels, both forces were aspects of a single interaction mediated by "four massless bosons". Through a subtle process called Spontaneous symmetry Breaking, three of the bosons acquired mass and became $W^{\pm}$ and Z^{0} particles with a consequent reduction in the range of what now appears as weak part of total interaction. The masses of $W^{\pm}$ and Z^{0} are regarded as attributes of the states they happen to be occupying rather than intrinsic attributes. The fourth electroweak boson, the photon remained massless and the range of electromagnetic part of total interaction accordingly stayed ∞.

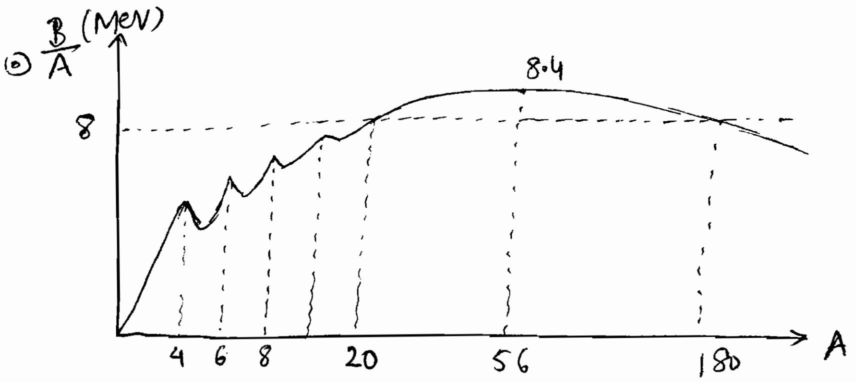
The discovery of $W^{\pm}$ and Z^{0} bosons has verified the theory.

- Binding Energy per Nucleon is low at

lower A : due to high surface energy. Smaller the nucleus, larger is the % of constituents at nuclear surface.

higher A : due to high coulumb repulsion.

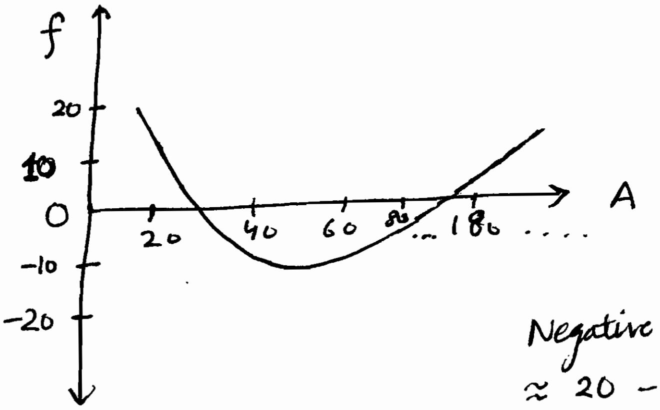
- More the B.E. ⇒ more the stability ⇒ more abundance ⇒ Iron abundant

- 

High stability at $A = 4, 8, 12, 16, 20 \dots$ indicates Pairing Energy i.e. even-even nucleons are more stable.

- Packing Fraction = $\left(\frac{M-A}{A}\right)$

A in a.m.u.
M is mass of nucleus



Negative for $\approx 20 - 180$

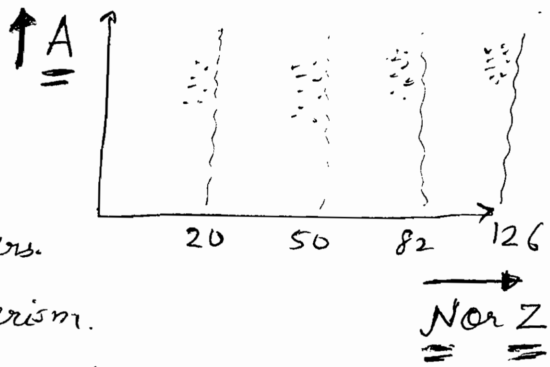
- Neutron should not have magnetic moment because it has zero charge. Actually magnetic moment depends on currents rather than charge. Eg. H atom has no net charge but does have a magnetic moment. Thus, neutron can have a magnetic moment if it contains equal (+)ve and (-)ve charges which are in motion.

(1) <u>Quark structure</u> : udd

(2) <u>Yukawa's Meson Structure</u> : According to Yukawa's particle exchange theory, neutron transforms into a proton and π meson for a fraction of time ($n \to p + \pi^-$). Since this meson is lighter than proton, its motion will be comparatively faster & resultant will be a net negative magnetic moment.

- Use of semi empirical mass formula include :
  1) accurate prediction of mass & Binding Energies of many nuclei
  2) Finding most stable nuclei in an Isobaric Series.
  3) Critical Fission Parameter calculation
  4) Validity of spontaneous reactions ($\Delta B.E. > 0$)
  5) Stability against β-decay

Nuclear Isomers



If nuclear isomers are plotted against proton number Z or neutron number N we get groupings just below magic numbers. These groupings are called Islands of Isomerism. There is a sharp break at magic numbers indicating disappearance of isomerism when a shell is completely filled.

Mass Conditions on β-decays

① β^- decay : ${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$

For β^- decay to take place : ${}^{A}_{Z}m > {}^{A}_{Z+1}m + {}^{0}_{-1}m$ (Nuclear Mass)

Adding Z electron masses $\boxed{{}^{A}_{Z}M > {}^{A}_{Z+1}M}$ (Atomic Mass)

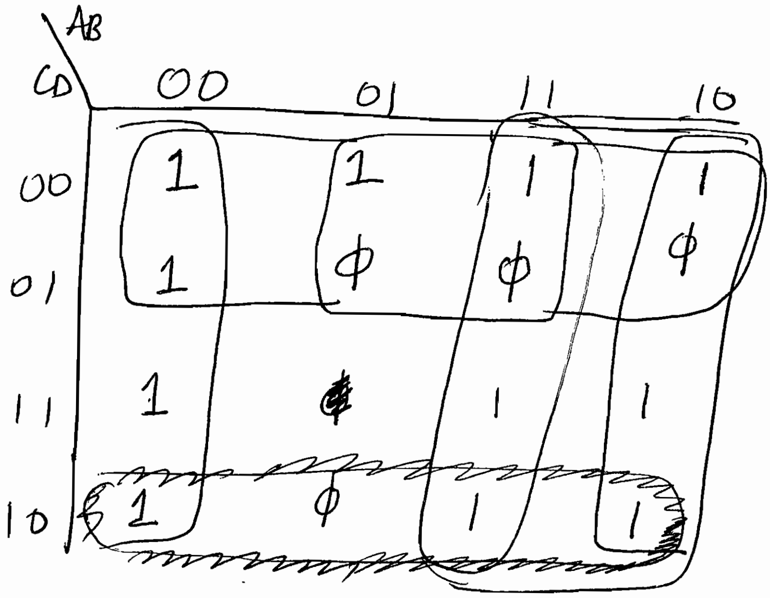
② β^+ decay : ${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e$

For β^+ to take place ${}^{A}_{Z}m > {}^{A}_{Z-1}m + {}^{0}_{+1}e$ (Nuclear Mass)

Adding Z electron masses $\boxed{{}^{A}_{Z}M > {}^{A}_{Z-1}M + 2\,{}^{0}_{+1}e}$ (Atomic Mass)

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	ϕ
0	1	1	0	ϕ
0	1	1	1	0
1	0	0	0	1
1	0	0	1	ϕ
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	ϕ
1	1	1	0	1
1	1	1	1	1

Such a problem would be extremely difficult for algebraic ways. Only karnough Map is a good option



$\overline{B} + \overline{C} + A$

(सारे 1s को include करना है, सारे ϕs को नहीं)

From Karnaugh Map

note the order

C \ AB	00	01	11	10
0	1		1	1
1	1		1	1

$A + \bar{B} = \underline{A + \bar{B}}$

(2)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

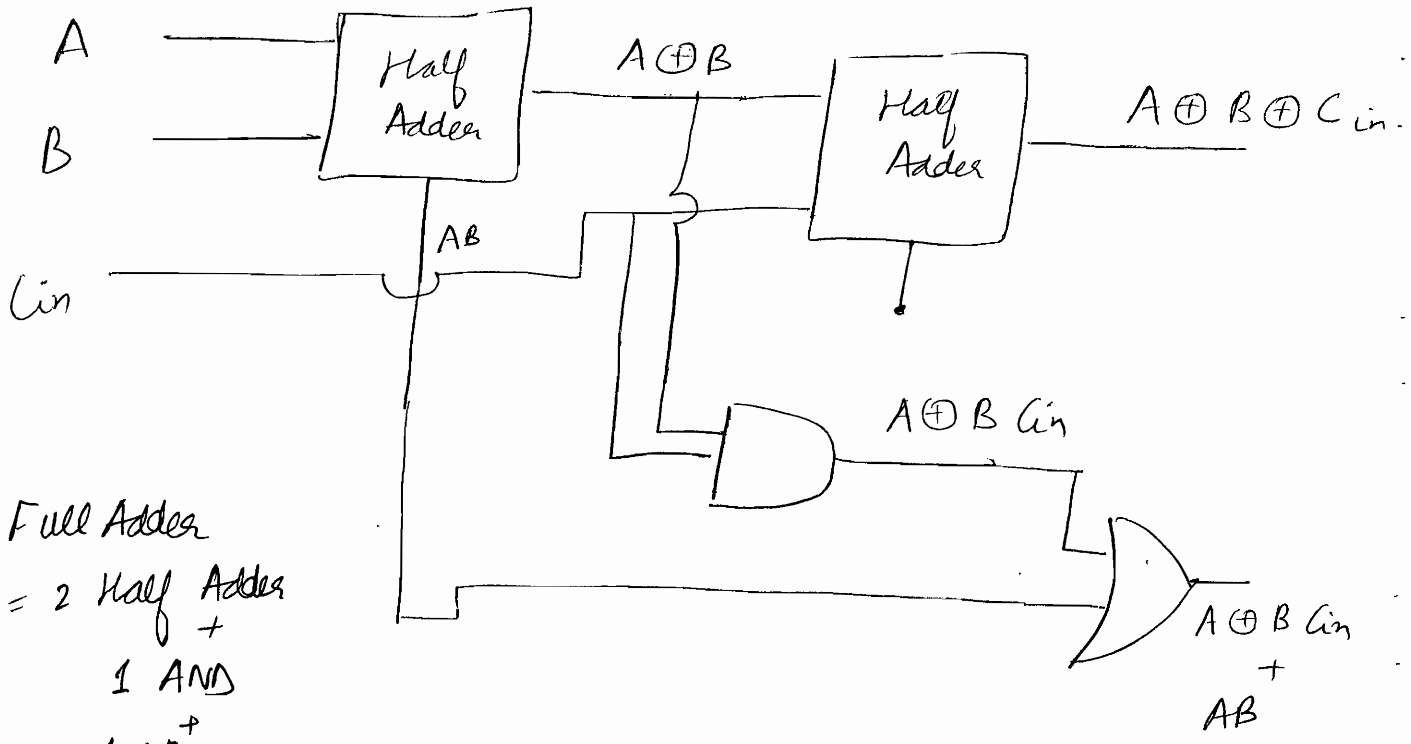
Simplifying this from algebraic identities will be hell of a task. Here karnaugh Map use is much simple.

CD \ AB	00	01	11	10
00		1	1	
01		1		1
11	1		1	1
10		1		

$$Y = \bar{B}CD + \bar{A}B\bar{D} + \bar{A}B\bar{C} + B\bar{C}\bar{D} + ACD + A\bar{B}D$$

(3) There is concept of don't-care term like the output of these combinations do not matter eg. these combination may never be possible inputs. For such combination, resultant can be take 0 or 1 according to suitability on karnaugh Map.

We can also represent Full Adder in terms of Half Adder



1 Full Adder
= 2 Half Adder + 1 AND + 1 OR

Some identities to reduce the functions have been mentioned. But all Combinational logic may not be so simple to directly apply identity. Hence, graphical method like **karnaugh Maps** are used.

For n literals, there are 2^n cells. 1 variable allowed to change in adjacent rows. In order to fill the karnaugh-matrix, in the expression to be minimized we must obtain each function to be minimized as a standard sum of products. For each term, corresponding 1 is entered into appropriate cell in the map. Now mark out common areas and write down corresponding functions:

eg. ① $F = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

From identities: $\bar{A}\bar{B}(\bar{C}+C) + A\bar{B}(\bar{C}+C) + AB(C+\bar{C})$

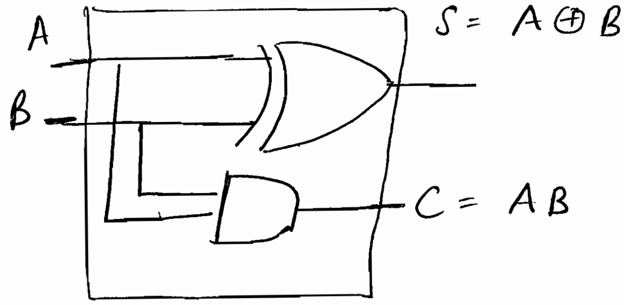
$= \bar{A}\bar{B} + A\bar{B} + AB = \bar{A}\bar{B} + A = \underline{A + \bar{B}}$

Half Adder : logic

A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

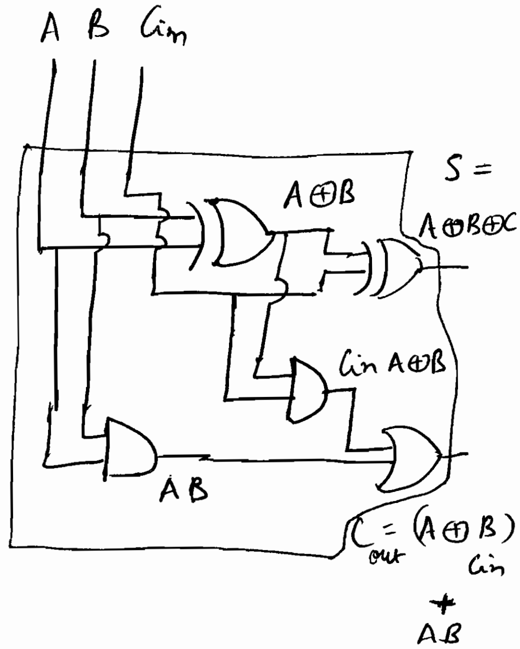
$\Rightarrow \quad C_{out} = AB$

$S = A \oplus B \quad \Longrightarrow$



Full Adder : logic

A	B	Cin	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$$S = \bar{A}\bar{B}\,C_{in} + \bar{A}B\,\overline{C_{in}} + A\bar{B}\,\overline{C_{in}} + AB\,C_{in}$$

$$= C_{in}\,\overline{(\bar{A}B + A\bar{B})} + \overline{C_{in}}\,(A \oplus B) = C_{in}\overline{(A \oplus B)} + \overline{C_{in}}\,(A \oplus B)$$

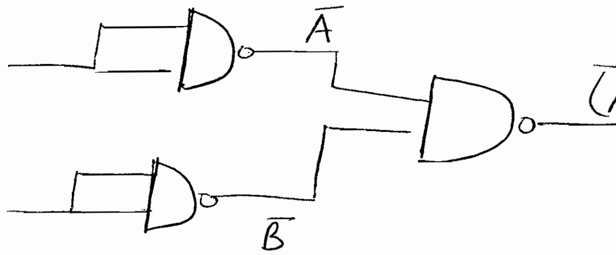
$$= A \oplus B \oplus C$$

$$C_{out} = \bar{A}B\,C_{in} + A\bar{B}\,C_{in} + AB\,\overline{C_{in}} + AB\,C_{in}$$

$$= \bar{A}B\,C_{in} + A\bar{B}\,C_{in} + AB$$

$$= C_{in}\,(A \oplus B) + AB$$

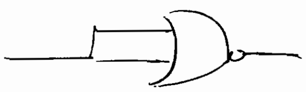
NAND → OR



$\overline{(\bar{A} \cdot \bar{B})} = A + B$

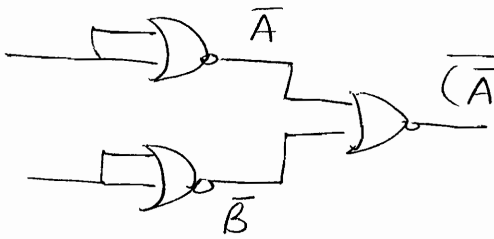
NOR

NOR → NOT



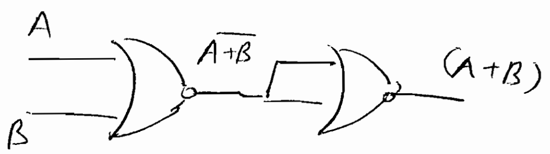
$\overline{A + A} = \bar{A}$

NOR → AND



$\overline{(\bar{A} + \bar{B})} = A \cdot B$

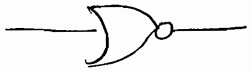
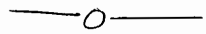
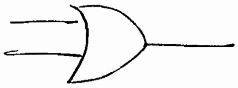
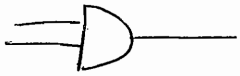
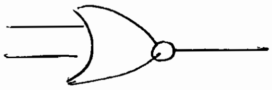
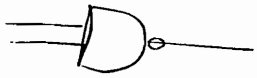
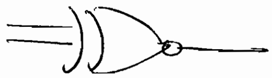
NOR → OR



Half Adder $Y = A + B$

Full Adder $Y = A + B + C$ ↑ carry

Fundamental Gates

1. NOT $Y = \bar{A}$  Or 
2. OR $Y = A + B$ 
3. AND $Y = A \cdot B$ 
4. NOR $Y = \overline{A+B}$ 
5. NAND $Y = \overline{A \cdot B}$ 
6. XOR $Y = A\bar{B} + \bar{A}B = A \oplus B$  (★)
7. X-NOR $Y = \overline{A\bar{B} + \bar{A}B} = \overline{A \oplus B}$  (★)

These gates can be realized by using diodes and transistors

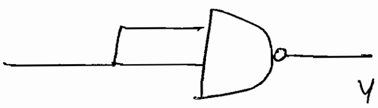
UNIVERSAL GATES

1. NAND
2. NOR

: Using any 1 of them any other logic gate can be realized.

NAND

NAND → NOT



$Y = (\overline{A \cdot A}) = \bar{A}$: Not Gate

NAND → AND

$Y = \overline{(\overline{A \cdot B}) \cdot (\overline{A \cdot B})}$

$= (A \cdot B) + (A \cdot B)$

$= A \cdot B$: AND gate

Boolean Identities

Two forms of identities

1. Sum of Products eg. $Y = AB + BC$
2. Product of Sums eg. $Y = (A+B)(C+D)$

Application of digital electronics

1. decision making (logic gates)
2. memory (flip flops)

Example of decision making $A + B > C$

A	B	C	Y	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	0	

For 1 : $Y = \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= (\bar{A}B + A\bar{B} + AB)\,\bar{C}$

For 0 : $\bar{Y} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

$Y = (A+B+C).(A+B+\bar{C}).(A+\bar{B}+\bar{C}).(\bar{A}+B+\bar{C}).(\bar{A}+\bar{B}+$

Write what is minimum.

Demorgan Laws

1. $\overline{(A+B)} = (\bar{A} \cdot \bar{B})$
2. $\overline{AB} = \bar{A} + \bar{B}$

How to Prove

A	B	(A + B)	$\overline{A+B}$	$\bar{A}$	$\bar{B}$	$\bar{A}\bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0
			✓			✓

Hence equal for $\overline{A+B}$

For three variables, $\overline{A+B+C} = \overline{A+(B+C)} = \bar{A} \cdot \overline{(B+C)}$

$= \bar{A} \cdot \bar{B} \cdot \bar{C}$

Hence valid for n literals.

★ For n literals, there are $2^{(2^n)}$ non-reducible functions

e.g. there can only be 16 functions of 2 binary variables A and B which are non-reducible.

Basic Identities

1. $A + \bar{A}B = A + B$

Proof L.H.S = $A + \bar{A}B$

$= A \cdot 1 + \bar{A}B$

$= A(1+B) + \bar{A}B$

$= A + AB + \bar{A}B$

$= A + (A+\bar{A})B$

$= \underline{A+B}$

R.H.S. $A + B$

$= A + B(A+\bar{A})$

$= A + AB + B\bar{A}$

$= \underline{A + \bar{A}B}$

$= LHS$

2. $(A+B)(A+C) = A + BC$

Proof $(A+B)(A+C) = AA + AC + BA + BC$

$= A + AC + BA + BC$

$= A(1+C) + AB + BC$

$= A + AB + BC$

$= A(1+B) + BC$

$= A + BC$

Now, $A_v(i + \beta\theta) = \theta$

$\Rightarrow i A_v = \theta[\ \cdot A_v\beta]$

$\Rightarrow \left(\frac{\theta}{i}\right) = A_{v_{eff}} = \left[\frac{A_v}{1 - A_v\beta}\right]$

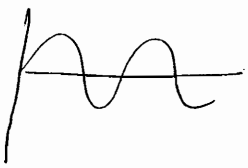
We need to increase $A_{v_{eff}} \Rightarrow A_v\beta \to 1$

[Berkhausen Criteria]

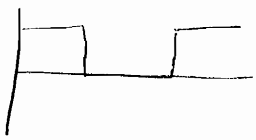
Hence $\boxed{A_v\beta = 1\angle 0^\circ}$ (phase shift = 0, 2π)

Digital Electronics

- Analog



- Digital



(1,0) binary digits

- literal → variable bits
  A, B, C [can take any value 0, 1]

- Boolean Algebra developed by George Bool.

(*)
$A + 1 = 1$
$A \cdot 0 = 0$
$A \cdot 1 = A$
$A + 0 = A$
$A + \bar{A} = 1$
$A \cdot \bar{A} = 0$

Basic identities

Basic Results

$A + (B \cdot C) = (A + B) \cdot (A + C)$

$A \cdot (B + C) = A \cdot B + A \cdot C$

$A + B = B + A$
$A \cdot B = B \cdot A$

$A + (B + C) = (A + B) + C$
$A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(*) For n literals, 2^n combinations

A	B
0	0
0	1
1	0
1	1

Absorption Results
$A + AB = A$
$A \cdot (A + B) = A$

Solid State Physics + Electronics

Analog Electronics

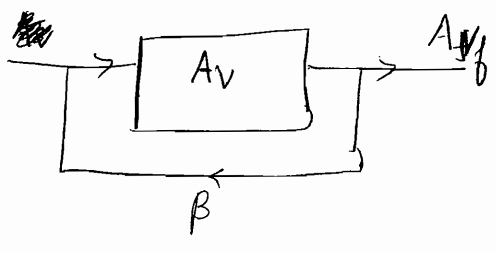
1. Physics of semiconductors
2. Transistor
   - Types
     - BJT (npn, pnp) (current driven)
     - FET
     - MOSFET [voltage driven]
   - Use
     - Amplifier
     - Oscillator
3. Opamp
4. Fundamental of microprocessors

Oscillator

Mechanism to create oscillations and sustain them. Oscillators are of two types: Harmonic (LC) and Anharmonic Oscillator. In Oscillators, some kind of output energy is feeback (⊕ve or ⊖ve) into system and whatever energy dissipation takes place, it is balanced.

Six types of oscillators include Colpitt $\left(\frac{1}{\sqrt{LC_{eq}}}\right)$, Hartley $\left(\frac{1}{\sqrt{L_{eq}C}}\right)$, Wein Bridge, RC-phase shift, Crystal Oscillator.

- Feedback Mechanism

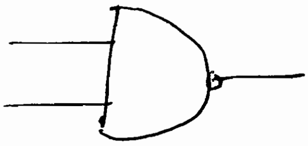


- Positive feedback increase the output change
- Negative feedback reduces the output change

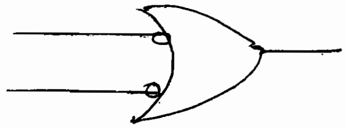
Realization of Gate Circuits

NAND Gate Implementation

Note that there are two way to represent NAND gate



AND-invert form

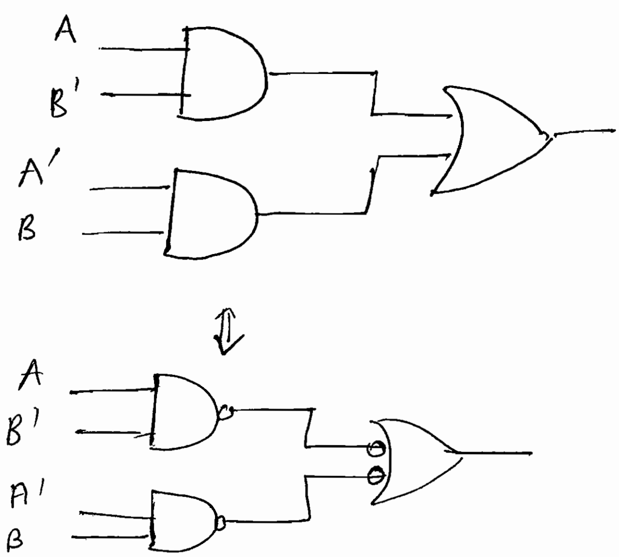


invert-OR form

So the general procedure is :

1. Make AND, OR, INVERT gate circuit
2. Replace AND by AND-invert NAND
   OR by invert-OR NAND
   NOT by multi-input NAND
3. Check all the bubbles. For every bubble that is not compensated by another bubble along the same wire, insert an invert (a single signal multi-input NAND) or complement the signal

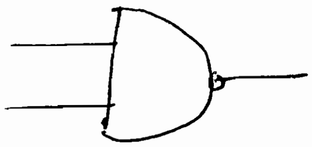
eg. $AB' + A'B$



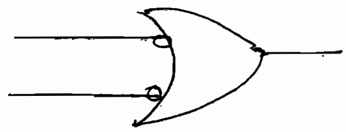
Realization of Gate Circuits

NAND Gate Implementation

Note that there are two way to represent NAND gate



AND-invert form

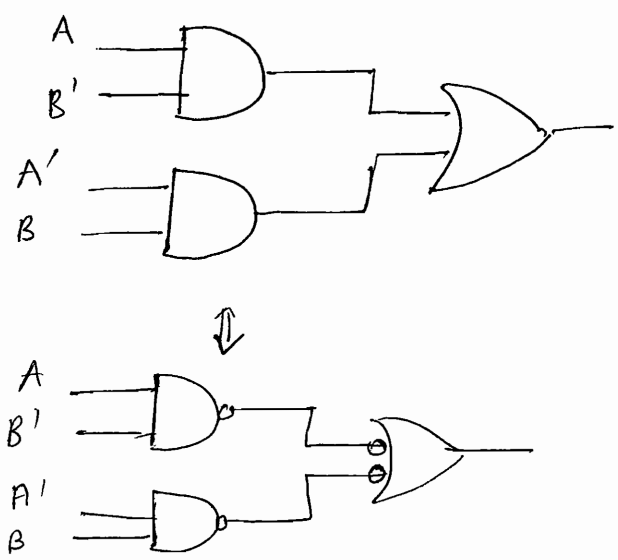


invert-OR form

So the general procedure is :

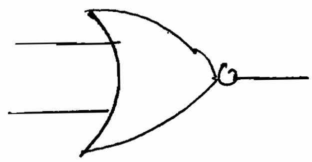
1. Make AND, OR, INVERT gate circuit
2. Replace AND by AND-invert NAND
   OR by invert - OR NAND
   NOT by multi-input NAND
3. Check all the bubbles. For every bubble that is not compensated by another bubble along the same wire, insert an invert (a single signal multi-input NAND) or complement the signal

eg. $AB' + A'B$



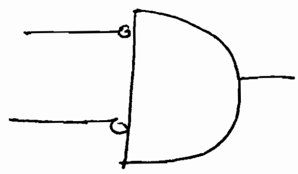
NOR Gate Implementation

NOR Gate is represented as



OR-invert

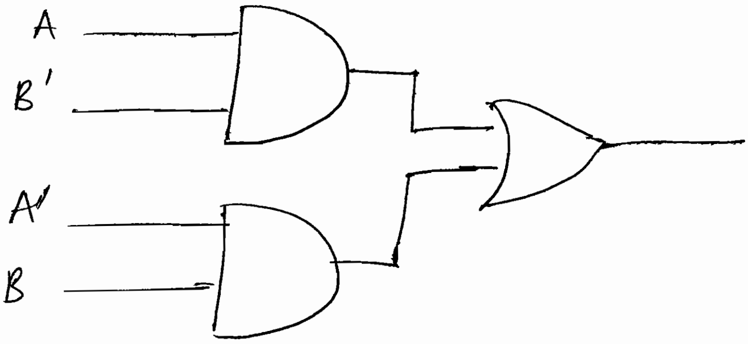
&



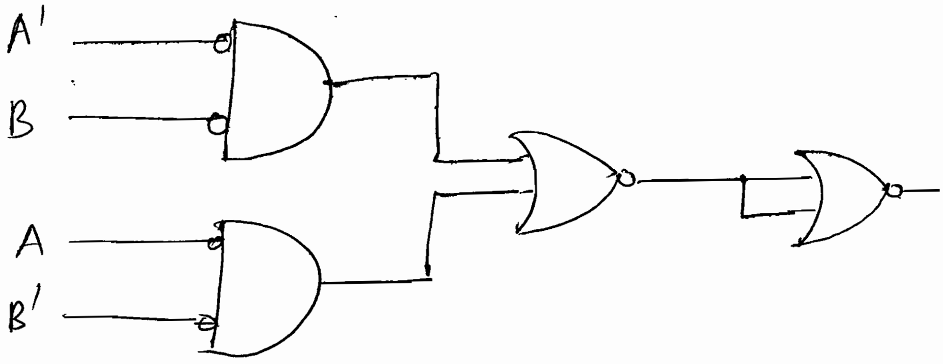
invert-AND

So the general procedure is same. Replace OR by OR-invert NOR, AND by invert-AND ~~NOR~~ and NOT by multi input NOR.

eg. $AB' + A'B$



⇕

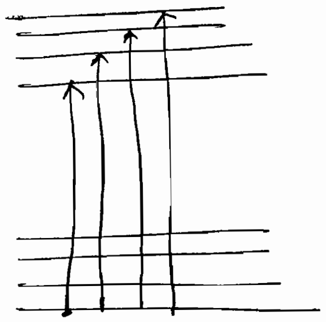


- Why are NAND and NOR called Universal Gates

Any digital system can be represented as combination of OR, AND and NOT gates. Since all the three can be easily implemented using NAND (or NOR), they are called universal gates.

(*) Fluoroscene is more powerful technique than Absorption

Absorption Spectra :



We obtain only 1 progression corresponding to $v'' = 0$

Fluorescence : By regulating Pressure of specimen gas, Collisions can be controlled. More Pressure ⇒ more collisions.

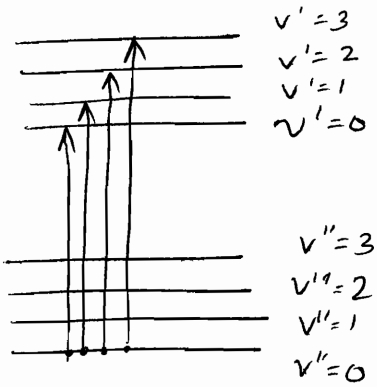
⇒ molecules lose or gain vibrational energy

⇒ [redacted] $v' = 0, 1, 2, 3, 4, \ldots$

⇒ As a result, fluorescence spectrum will consist of many progressions corresponding to various v' values. With sufficient pressure, entire band system can be obtained in fluoroscent spectrum.

(*) Fluoroscene in solutions

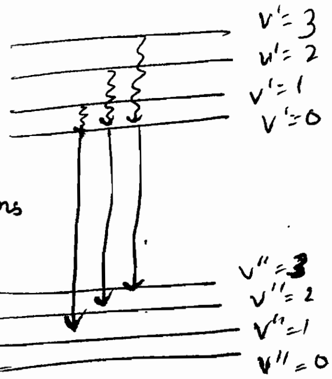
Absorption Spectra :



- Absorption spectra corresponding to $v'' = 0$ and various v' values.
- lowest frequency band is (0,0)

Fluorescent Spectra :

- Before fluorescent emission, excited molecules lose vibrational energy due to collisions and come down to $v' = 0$ of excited electronic state. This <u>non radiative process</u> is called VIBRATIONAL DEACTIVATION. Takes place in less than 10^{-4} seconds after excitation.



- Fluorescent spectra corresponding to $v' = 0$ and various v'' values
- highest frequency band is (0,0)

(*) Difference betwee Raman and Fluorescence

(1) Fluoroscence : Absorption of Photon, then emission of photon

Raman : Scattering of Photon with changed energy

(2) Frequency of Fluoroscene always (almost) lower than excited. (exceptions are heavier molecules like I_2)

Frequency of Raman both lower & greater than excited.

(3) Fluoroscence occurs when frequency of incident light coincides with one of absorption frequencies of [redacted] gas molecules.

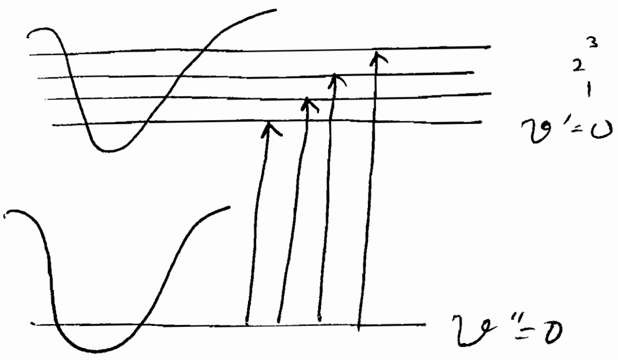
Raman Effect can take place for 'any' frequency of incident light i.e. incident frequency is not related to characteristic frequency of specimen molecules.

(4) In fluoroscence, transition between different electronic states while in Raman effect its difference in vibrational and rotational levels.

(*) For studying molecules, absorption band is better than emission band

(1) Appreciable no. of molecules are available in ground state.

(2)



First differences

$\bar{\omega}_e' - 2\bar{\omega}_e' x_e$

$\bar{\omega}_e' - 4\bar{\omega}_e' x_e$

$\bar{\omega}_e' - 6\bar{\omega}_e' x_e$

⇒ we can get $\bar{\omega}_e$, $\bar{\omega}_e x_e$, k, Dissociation energy by extrapolation

For emission spectra, complex progressions from $v' = 0$ to v'', $v' = 1$ to v'' etc

This is because, the emission source of band spectra are usually at temperature much higher than room temperature, & so a large no. of vibrational levels ($v'' = 0, 1, 2, 3 \ldots$) are appreciably populated.

Importance of 21 cm line

1. Only way to observe cold (therefore dark) interstellar medium since neutral H is abundant

2. Gives a map of H present in galaxy b'coz it can penetrate dust clouds.

3. From doppler shift, we can measure speed of rotation of galaxies & how they interact with each other.

4. From intensity of lines, we can measure mass distribution in galaxies and fix up limits of gravitational constant G.

5. Its only way to probe into "dark ages" from recombination to reionization as the H atom which had ionized at that time will appear as holes in 21 cm background.

Q) Derive Dirac's formula to calculate the energy perturbation.

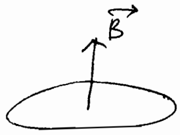
Solving the H atom problem quantum mechanically, with potentials taken as Coulumb Potential, the energy values obtained are

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{Z^2}{n^2}$$

But these energy levels failed to explain fine structure of H spectrum.

Dirac considered 2 perturbations in order modify the energy values:

SPIN ORBIT INTERACTION



$$B_{int} = \frac{\mu_0}{4\pi}\left(\frac{e}{m}\right)\frac{\vec{L}}{r^3}$$

$$\Delta E_{l,s} = -\vec{\mu}_s \cdot \vec{B}$$

$$= \frac{\mu_0}{4\pi}\left(\frac{e}{m}\right)^2 \frac{\vec{L}\cdot\vec{S}}{r^3}$$

Using averages of $\left(\frac{1}{r^3}\right)$ from Radial eigenfunction, we get

$$\Delta T_{l,s} = -\frac{R_\infty \alpha^2 Z^4}{n^3}\left[\frac{j(j+1) - l(l+1) - s(s+1)}{2l(l\pm\frac{1}{2})(l+1)}\right]$$

α : fine structure const

R_∞ : Rydberg's constant

RELAVISTIC CORRECTION

Hamiltonian is given by

$$H = k + V$$

Taking k as $E - m_0c^2$, we get

$$H = \frac{1}{2}m_0v^2 + \frac{3}{8}\frac{m_0v^4}{c^2} + \ldots + V$$

Hence perturbation of $-\frac{3}{8}\frac{m_0v^4}{c^2}$

Taking this into account, we get

$$\Delta T_l = +\frac{R_\infty \alpha^2 Z^4}{n^3}\left[\frac{1}{l+\frac{1}{2}} - \frac{3}{4n}\right]$$

Now total perturbation

$$= \Delta T = \Delta T_l + \Delta T_{l,s}$$

using $j = l \pm \frac{1}{2}$, we get

$$\Delta T = -\frac{R_\infty \alpha^2 Z^4}{n^3}\left[\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right]$$

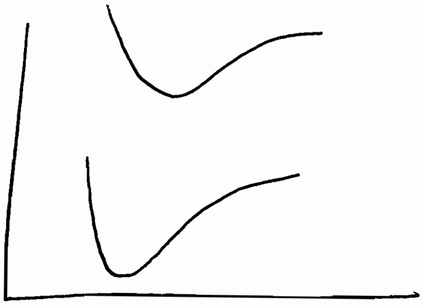
$$R_\infty \alpha^2 = 5.84\ cm^{-1}$$

Fine structure constant : $\alpha = \left[\frac{e^2}{(4\pi\varepsilon_0)\hbar c}\right]$

Rydberg's Constant : $R_\infty = 1.09 \times 10^7\ m^{-1}$

Electronic Vs Vibrational Rotational

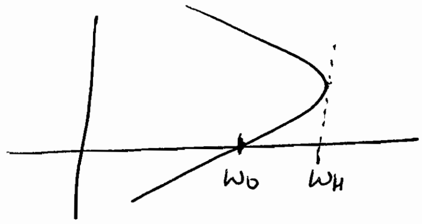
Electronic	Vibrational Rotational
(1) Emission as well as Absorption mode	Absorption mode studies only
(2) Homonuclear also	Homonuclear does not show spectra
(3) Red or Violet degraded ($B'-B'' <$ or >0) as Δv = different	Red degraded only (B' $B'' < 0$)
(4) Strong tendency of head formation	Weak tendency of head formation



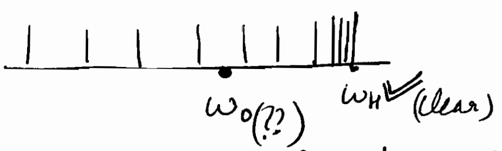
Higher electronic energy curve
- more r_e due to loosely bound atoms in a higher energy state
- shallower & broader

Importance of Q branch

normally :

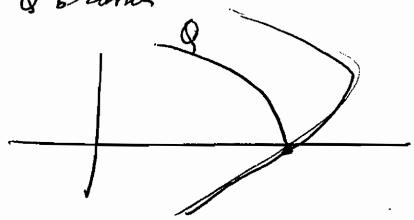


w_H : Band head
w_0 : Band Origin



difficult to find band origin
Usually assumed as w_H.

If Q branch



Head at $J'' = -\frac{1}{2}$
which is very close to Band Origin
Head Q_{Head} is used to obtain Band Origin

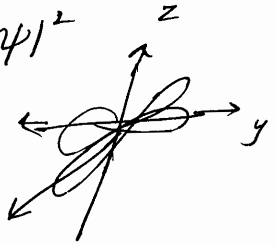
Q When $\vec{J}$ precess about $\vec{B}$, $\vec{L}$ and $\vec{S}$ have $m_l \hbar$ and $m_s \frac{\hbar}{2}$ components along $\vec{B}$ or $\vec{J}$??

Q Significance of multiplicity $r = 2$ for $l = 0$ if the number of values of j is only 1 i.e. $j = \left(\frac{1}{2}\right)$.

Q ~~lecture 2 second last page 4 lines~~ ??

Q If in $\psi(r, \theta, \phi)$ $\Phi = e^{im\phi}$
no dependence on ϕ in $|\psi|^2$
Then how comes p_x and p_y ?

actually doughnut shaped



Q कब क्या होगा ? When will Raman Scattering and near infrared spectrum take place ??

उस energy की wave कौन !!