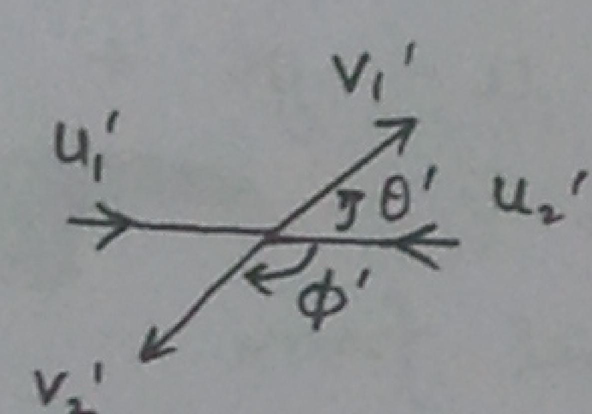
Formulas to remember ①

- $\vec{v} = \dot{r}\,\hat{r} + \dot{\theta} r\,\hat{\theta}$
- $\vec{a} = (\ddot{r} - \dot{\theta}^2 r)\,\hat{r} + (2\dot{r}\dot{\theta} + \ddot{\theta} r)\,\hat{\theta}$
- $U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r}$
- $R_{CM} = \dfrac{\sum m_i r_i}{\sum m_i}$
- For an elastic collision as observed from COM frame,

$|v_1'| = |u_1'|$
$|v_2'| = |u_2'|$

i.e. magnitude of velocities do not change from COM frame of reference, only change in direction.

$\theta' + \phi' = \pi$



$$\tan\theta = \frac{\sin\theta'}{\cos\theta' + \left(\frac{m_1}{m_2}\right)}$$

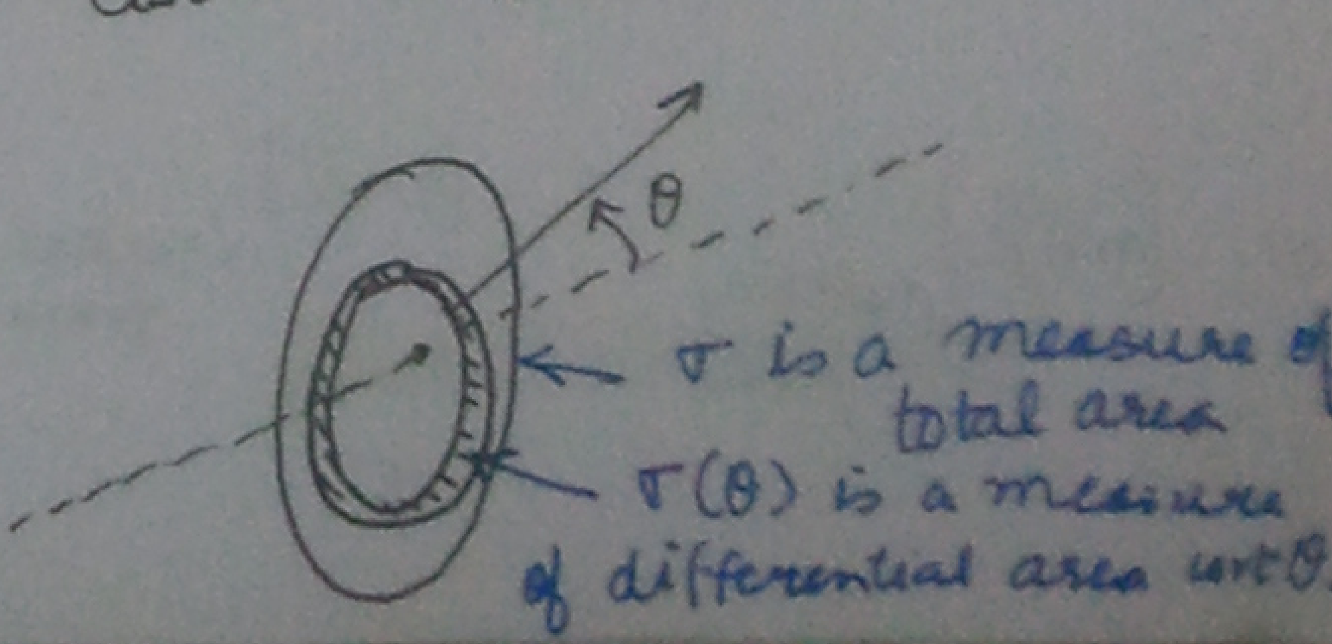
$$\phi = \left(\frac{\phi'}{2}\right)$$

- Energy is transformed in the same way other vectors are transformed wrt. COM Transformation.

$$E' = E - E_{CM}$$

- $\sigma = \displaystyle\int_{\theta=0}^{\theta=\pi} \sigma(\theta)\; 2\pi \sin\theta \, d\theta$

$d\Omega = 2\pi \sin\theta \, d\theta$



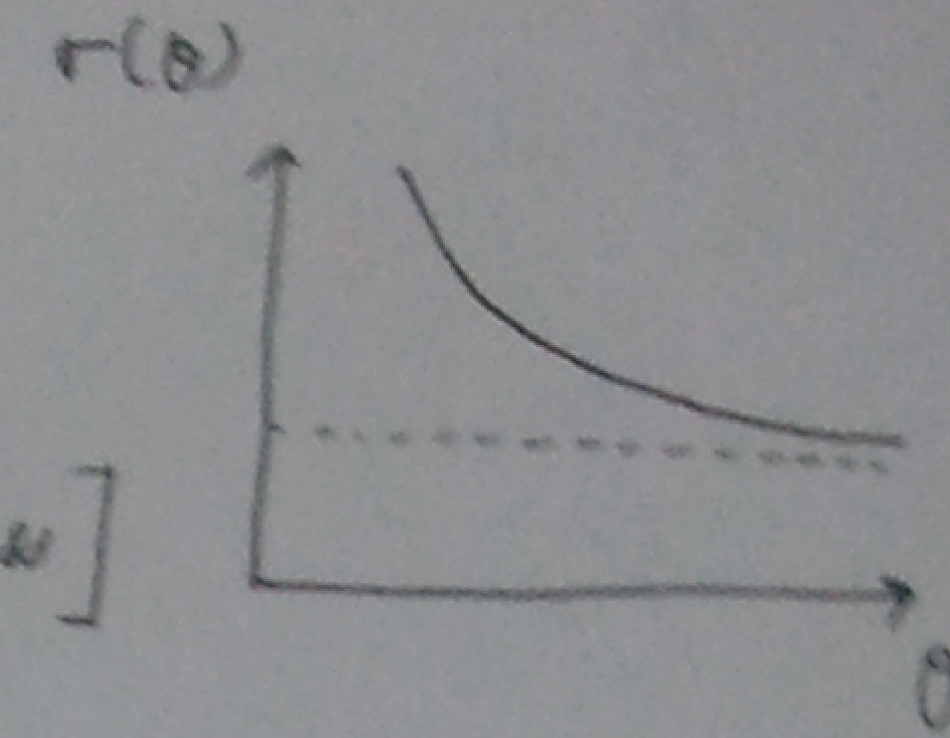
- $\sigma(\Omega) = \left[\dfrac{dN}{I\,d\Omega}\right]$ and $\sigma = \left[\dfrac{N}{I}\right]$
- $\sigma(\theta) = -(b/\sin\theta) * (db/d\theta)$

⊙ For Rutherford Scattering,

$$b = \frac{C_1}{2E_\alpha} \cot\left(\frac{\theta}{2}\right)$$ where $C_1 = \left[\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\right]$ i.e. F·

Corresponding

$$\sigma(\theta) = \frac{C_1^2}{16 E_\alpha^2} \operatorname{cosec}^4\left(\frac{\theta}{2}\right)$$



[All parameters wrt. Centre of m_2 or Centre of Mass]

⊙ For hard - sphere scattering

$$\sigma(\theta) = \left[\frac{R^2}{4}\right] \quad ; \quad \sigma = \pi R^2$$

⊙ Rocket Motion : $v = u - gt + u_{exhaust} \ln\left(\frac{M_0}{M}\right)$

Thrust = $-\left(\frac{dM}{dt}\right) \vec{u}_{exhaust}$; [redacted] P TOTAL

⊙ For observation in non intertial frame

$$[F]_{\substack{\text{observed in}\\ \text{non inertial}\\ \text{frame}}} + F_{pseudo} = m\,[a]_{\substack{\text{observed in}\\ \text{non inertial}\\ \text{frame}}}$$

$$\vec{a} = \left(\frac{d\vec{p}}{dt}\right)$$ by definition irrespective of frame of refe

⊙ For rotating vector $\vec{\lambda}$ $\qquad \frac{d\vec{\lambda}}{dt} = [\vec{\omega} \times \vec{\lambda}]$

⊙ Transformation from rotating frame to inertial frame,

$$\frac{d}{dt}\vec{\lambda} = \left[\frac{d}{dt}\vec{\lambda}\right]_{\substack{\text{observed}\\ \text{in rotating}\\ \text{frame}}} + [\vec{\omega} \times \vec{\lambda}]$$

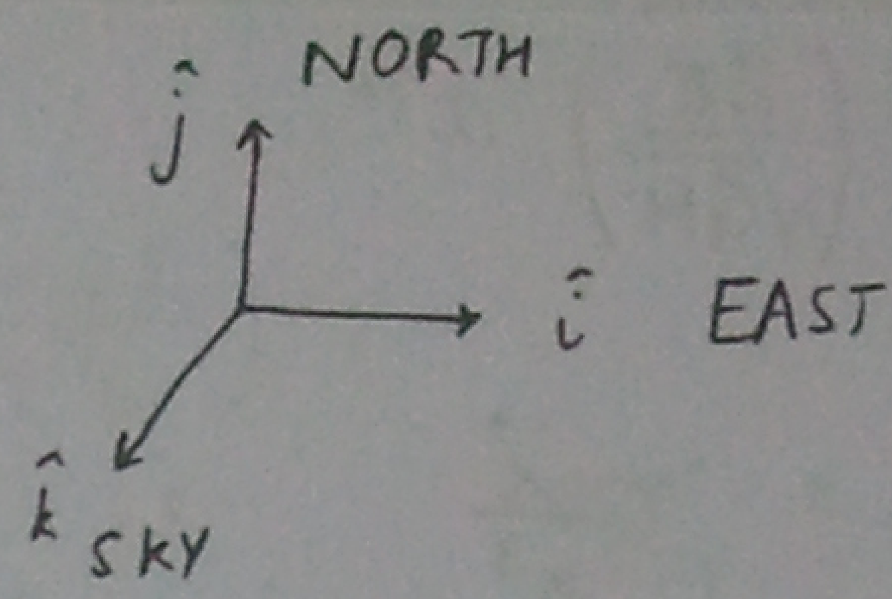
⊙ $$[m\vec{a}]_{\substack{\text{rotating}\\ \text{frame}}} = [m\vec{a}]_{\substack{\text{inertial}\\ \text{frame}}} + \underbrace{[-2m(\vec{\omega} \times \vec{v}')]}_{\text{Coriolis}} + \underbrace{[-m\,\vec{\omega} \times (\vec{\omega}\times}_{\text{Centrifugal}}$$

or

F

v': velocity measured in rotating frame

Formulas to remember ②

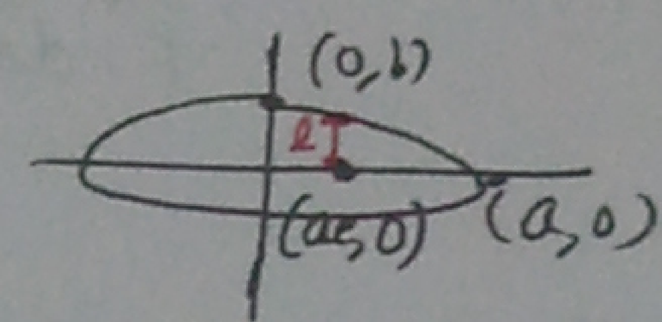


⊙ $\vec{\omega} = \omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}$

$F_{coriolis} = -2m\,(\vec{\omega}\times\vec{v})$

$F_{centrifugal} = -m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})$

⊙ Ellipse : $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$



$\Leftrightarrow$

$\dfrac{l}{r} = (1 + e\cos\theta)$ $l = \left(\dfrac{b^2}{a}\right) = a(1-e^2)$

$b^2 = a^2(1-e^2)$

$r_{max} = \dfrac{l}{1-e} = a(1+e)$

$r_{min} = \dfrac{l}{1+e} = a(1-e)$

⊙ $\underline{e}$ $\underline{\text{Conic}}$

0 : circle

$0<e<1$: ellipse

1 : Parabola

$e>1$: Hyperbola

⊙ $\left(\dfrac{dA}{dt}\right) = \left(\dfrac{J}{2\mu}\right) = \text{Const.}$

⊙ Trajectory of Motion under Central Force :

$$\frac{d^2u}{d\theta^2} + u = -\left[\frac{\mu\,F(u)}{J^2u^2}\right]$$

always J^2 μ^1

⊙ For $F = -(k/r^2)$, the trajectory is

$$\left\{\frac{\left[\frac{J^2}{k\mu}\right]}{r} = 1 + \underbrace{\sqrt{1 + \frac{2EJ^2}{\mu k^2}}}_{e}\cos(\theta-\theta_0)\right.$$

$v_{esc} = \sqrt{\dfrac{2GM}{r}}$ [in general]

$v_0 = \sqrt{\dfrac{GM}{r}}$ [for circular case] ...... [redacted]

$$T = \left(\frac{2\pi}{\sqrt{GM}}\right) a^{3/2}$$

- $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$; $F = -\nabla U$ and $E = -\nabla V$

  $U = -\int_{\infty}^{r} F \cdot dr$ and $V = -\int_{\infty}^{r} E \cdot dr$

  $\vec{F} \cdot d\vec{r} = -dU$ and $\vec{E} \cdot d\vec{r} = -dV$

- Gravitational self Energy for a sphere $= -\frac{3}{5}\left(\frac{GM^2}{R}\right)$

- Gauss law : $\oint_s E \cdot ds = (-4\pi G)\, m_{enclosed}$

  or

  $\nabla \cdot \vec{E} = -4\pi\rho G$

- $\vec{J} = \sum_i \vec{r}_i \times \vec{p}_i$

- $$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

  $I_{xx} = \sum m_i (x_i^2 + y_i^2)$

  $I_{xy} = -\sum m_i (x_i y_i)$

  $I_{xz} = -\sum m_i (x_i z_i)$

- $K.E. = \frac{1}{2}\vec{\omega} \cdot \vec{J} = \frac{1}{2}\vec{\omega} \cdot [\overleftrightarrow{I}\, \vec{\omega}]$

- Torque free Motion of a symmetric body :

  $\rightarrow \frac{\omega_1^2}{\left(\frac{2k}{I_1}\right)} + \frac{\omega_2^2}{\left(\frac{2k}{I_2}\right)} + \frac{\omega_3^2}{\left(\frac{2k}{I_3}\right)} = 1$ POINSOT EQUATION

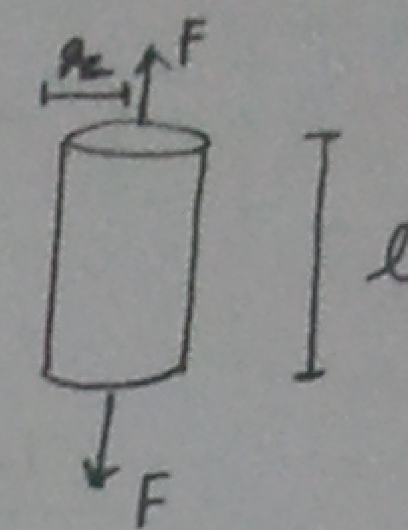
  $\rightarrow \vec{\omega} = A \sin \Omega t\, \hat{i} + A \cos \Omega t\, \hat{j} + \omega_3 \hat{k}$ where $\Omega = \omega_3\left(1 - \frac{I_3}{I_1}\right)$ [Precession Velocity]

Formulas to remember (3)

- Precession Motion of a Top:

  Precessional Angular Velocity $\Omega = \left(\dfrac{mgl}{I\omega}\right)$

- Lateral Change in length $dr = \left(\dfrac{\sigma P}{Y}\right) r$



- $Y = 3K(1-2\sigma)$

- $Y = 2\eta(1+\sigma)$

- $\eta = \dfrac{(F/A)}{(dv/dz)}$

- Stokes law: $F = 6\pi\eta r v$: Frictional drag on a spherical body

- Terminal velocity $= \dfrac{2r^2 g(\rho-\sigma)}{9\eta}$

- $R = \left(\dfrac{\rho v D}{\eta}\right)$ Critical Value: 1000

- Poiselles Formula for flow in Capillaries $Q = \left(\dfrac{\pi P a^4}{8\eta l}\right)$

- Bernouli Theorem: $\frac{1}{2}\rho v^2 + P + \rho g h = \text{Const.}$

- Venturimeter: $Q = c\sqrt{h}$ where $c \propto \sqrt{\dfrac{A_1^2 A_2^2}{A_1^2 - A_2^2}}$

- Lorentz Transform:
  $x' = \alpha(x - vt)$
  $t' = \alpha\left(t - \dfrac{v}{c^2}x\right)$

- Length Contraction $\Rightarrow L = \left(\dfrac{L_0}{\alpha}\right) \Rightarrow L_0$: Proper length is largest

- Time Dilation $\Rightarrow T = \alpha T_0 \Rightarrow T_0$: Proper Time is least

Composition of Velocities

$$u_x' = \frac{u_x - v}{1 - u_x \frac{v}{c^2}}$$

$$u_y' = \frac{u_y}{\alpha\left(1 - u_x \frac{v}{c^2}\right)}$$

$$u_z' = \frac{u_z}{\alpha\left(1 - u_x \frac{v}{c^2}\right)}$$

Space-Time Coordinates : $(x, y, z$

Minkowsky's 4-d Coordinates

(x_1, x_2, x_3, x_4)

[Also called 4-vector]

where

$x_1 = x$

$x_2 = y$

$x_3 = z$

$x_4 = ict$

Space Time Interval is defined as interval in Minkowsky's Coordinates i.e.

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 + i^2c^2(t_1 -$$

$$= (dx)^2 + (dy)^2 + (dz)^2 - c^2(dt)^2$$

Velocity 4-vector or Four Velocity

$$= \begin{bmatrix} \alpha v_x \\ \alpha v_y \\ \alpha v_z \\ i\alpha c \end{bmatrix}$$

Momentum 4-vector or Four Momentum $= m_0 \begin{bmatrix} \text{velocity} \\ \text{Four vector} \end{bmatrix}$

$$= \begin{bmatrix} \alpha m_0 v_x \\ \alpha m_0 v_y \\ \alpha m_0 v_z \\ i\alpha m_0 c \end{bmatrix} = \begin{bmatrix} m v_x \\ m v_y \\ m v_z \\ i mc \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ i\frac{E}{c} \end{bmatrix}$$

Minkowsky's Transformation Matrix:

$$\begin{bmatrix} \alpha & 0 & 0 & i\alpha\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\alpha\beta & 0 & 0 & \alpha \end{bmatrix}$$

$$m' = \frac{m\left(1 - \frac{u_x v}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p_x' = \left(\frac{p_x - \frac{Ev}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$$

since $x = ct$ & $E = c$
$x \Leftrightarrow E$, $t \Leftrightarrow p$
Similar transformation as in Lorentz or us Minkowsy

$$E' = \left(\frac{E - vp_x}{\sqrt{1 - v^2/c^2}}\right)$$

Formulas to remember ④

$E = mc^2$

1 a.m.u	:	931 MeV
1 e^-	:	0.51 MeV
1 Proton	:	938 MeV
1 Neutron	:	940 MeV

Rest Mass Energies $= m_0c^2$

NEUTRON is heaviest

$$E^2 = (pc)^2 + (m_0c^2)^2$$

$$\gamma' = \gamma\sqrt{\frac{1-\left(\frac{v}{c}\right)}{1+\left(\frac{v}{c}\right)}} \quad \text{or} \quad \gamma' = \gamma\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

$$\rho_0' = \alpha^2 \rho_0$$

Aberration : $$\tan\theta = \left(\frac{\tan\theta'}{1+\frac{v}{c\cos\theta'}}\right)$$

Rocket Motion

$$M = M_0 - \alpha t$$

$$M = M_0(1-\beta t)$$